Local gravitational instability of stratified rotating fluids

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Based on *Nipoti (2023), Nipoti, Caprioglio & Bacchini (2024), Bacchini, Nipoti et al. (2024)*

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WLM dwarf (credits: Jang et al)

Let's start from non-rotating stratified fluids

- Is Jeans criterion valid when the unperturbed medium is inhomogeneous?
- Condition for local perturbation $\lambda \ll L$
- $\lambda =$ perturbation wavelength
- $L =$ macroscopic scale $(L = p/||\nabla p||$ or $L = \rho/||\nabla \rho||)$

Jeans wavelength in non-rotating self-gravitating fluid

- $L = \dfrac{p}{||\nabla p||} = \dfrac{p}{\rho||\nabla\Phi||} \approx \dfrac{c_s^2 L^2}{GM} \approx \dfrac{c_s^2 L^2}{G\rho L^3} = \dfrac{c_s^2}{G\rho L}$

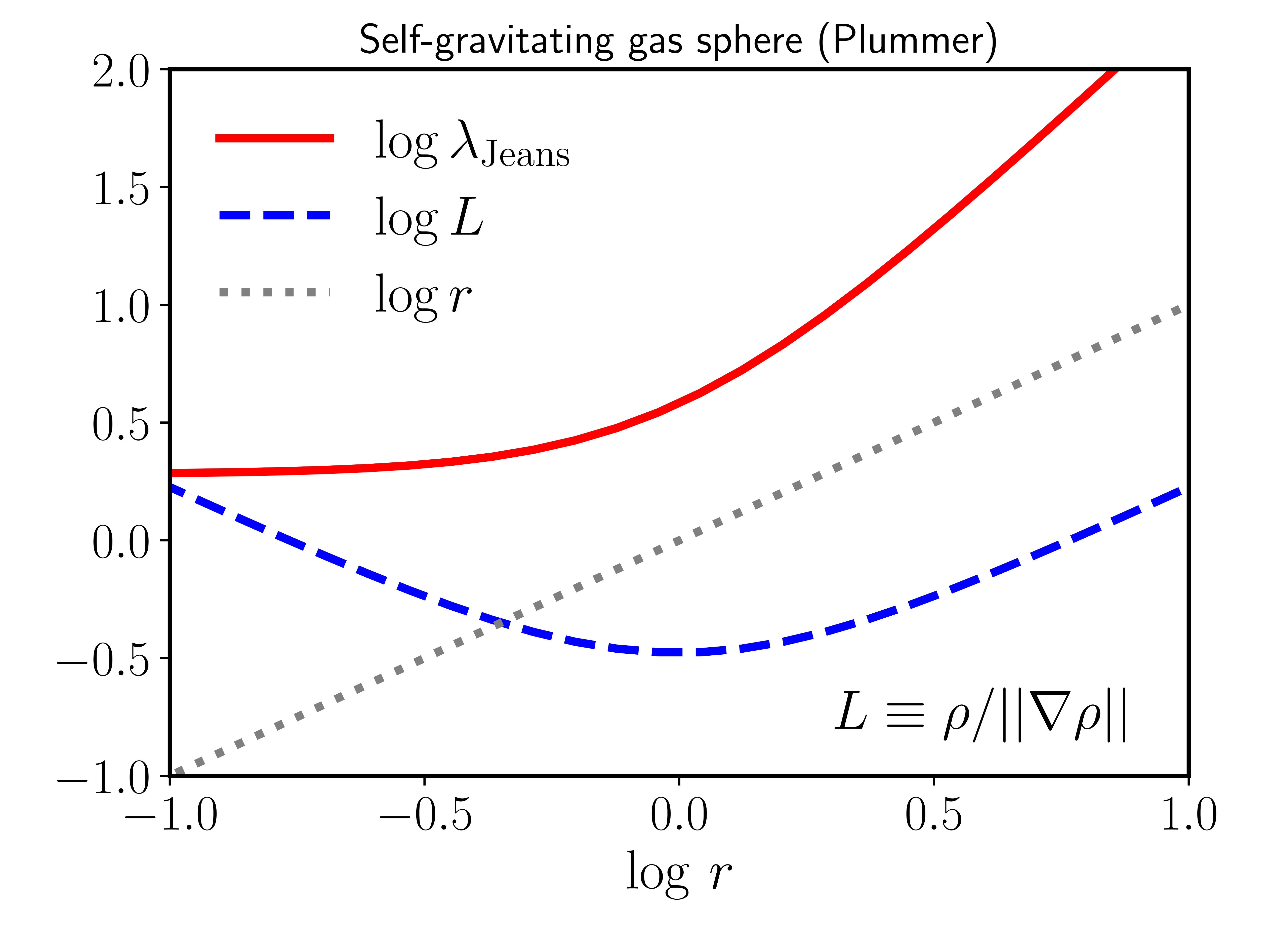
- $L^2 \approx \dfrac{c_s^2}{G\rho} \approx \lambda_{\rm Jeans}^2$

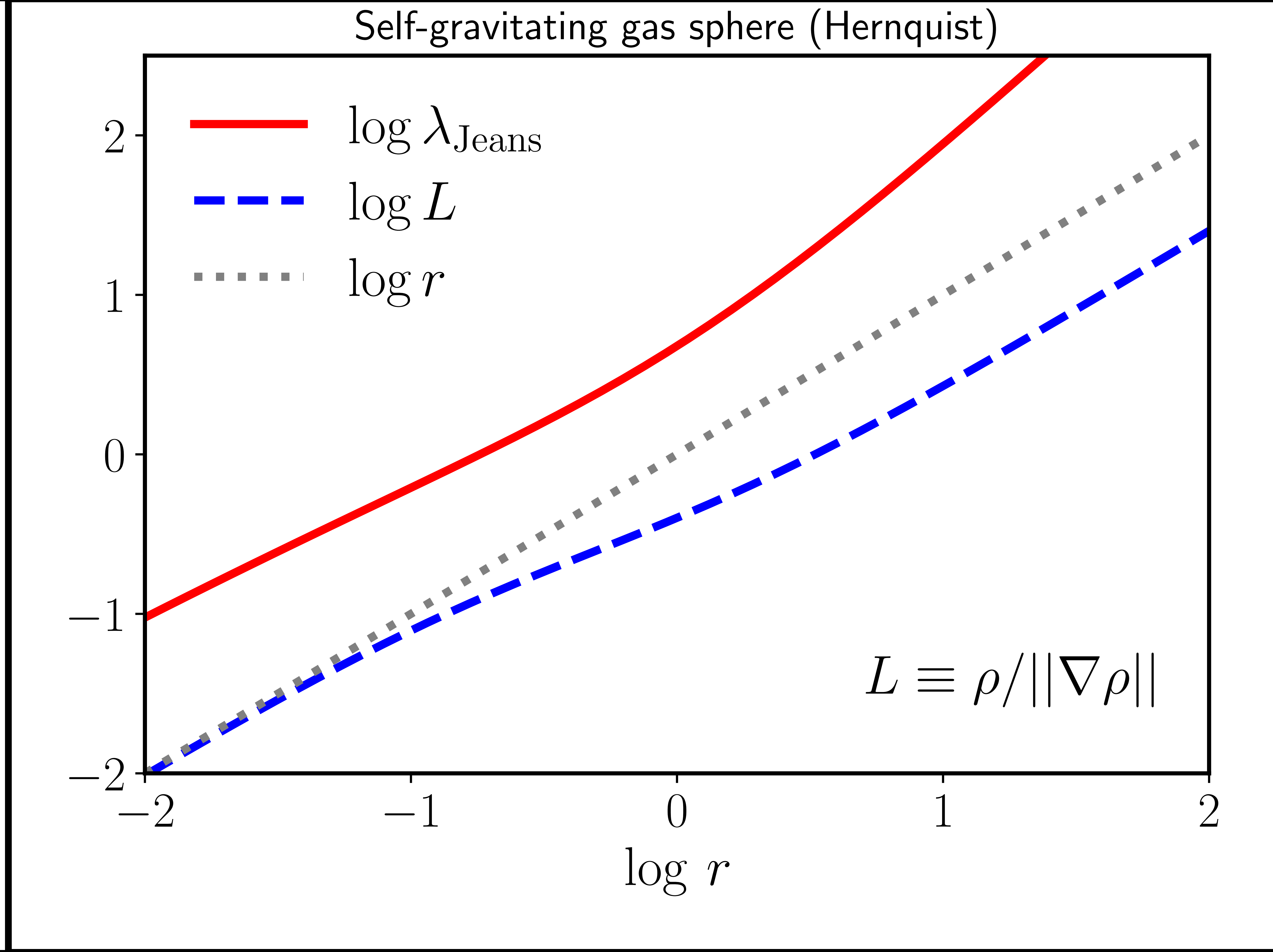
- The Jeans criterion proves that perturbations with $\lambda \ll L$ are stable, but is inconclusive about perturbations with $\lambda \gtrsim L$

Bertin (2014) "Dynamics of galaxies" (page 132)

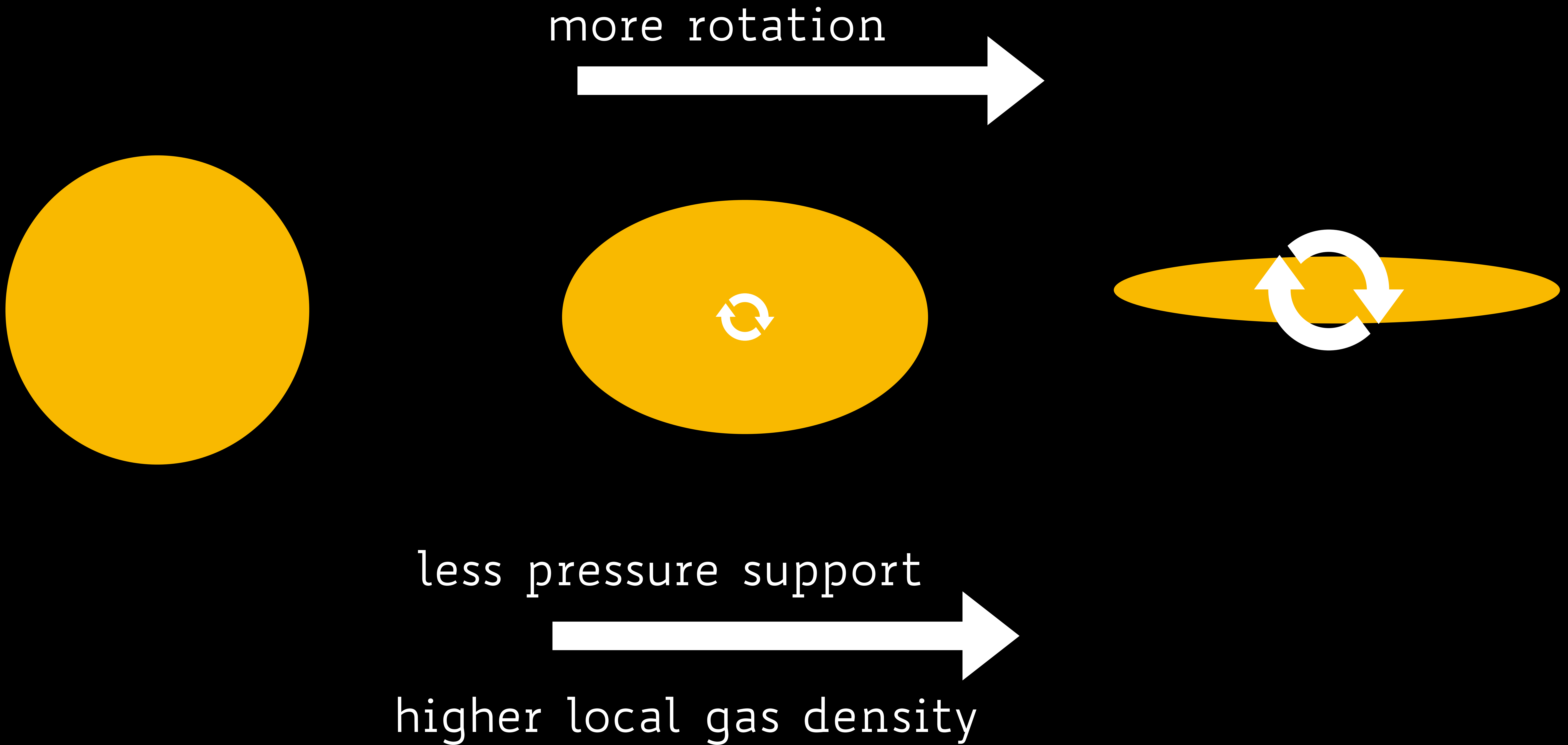
We have thus proved that the system is not large enough to accommodate Jeans unstable perturbations. In other words, it appears that a finite spherical, nonrotating, self-gravitating system has its size determined in such a way that the Jeans instability is suppressed.

Non-rotating stratified fluids are locally stable



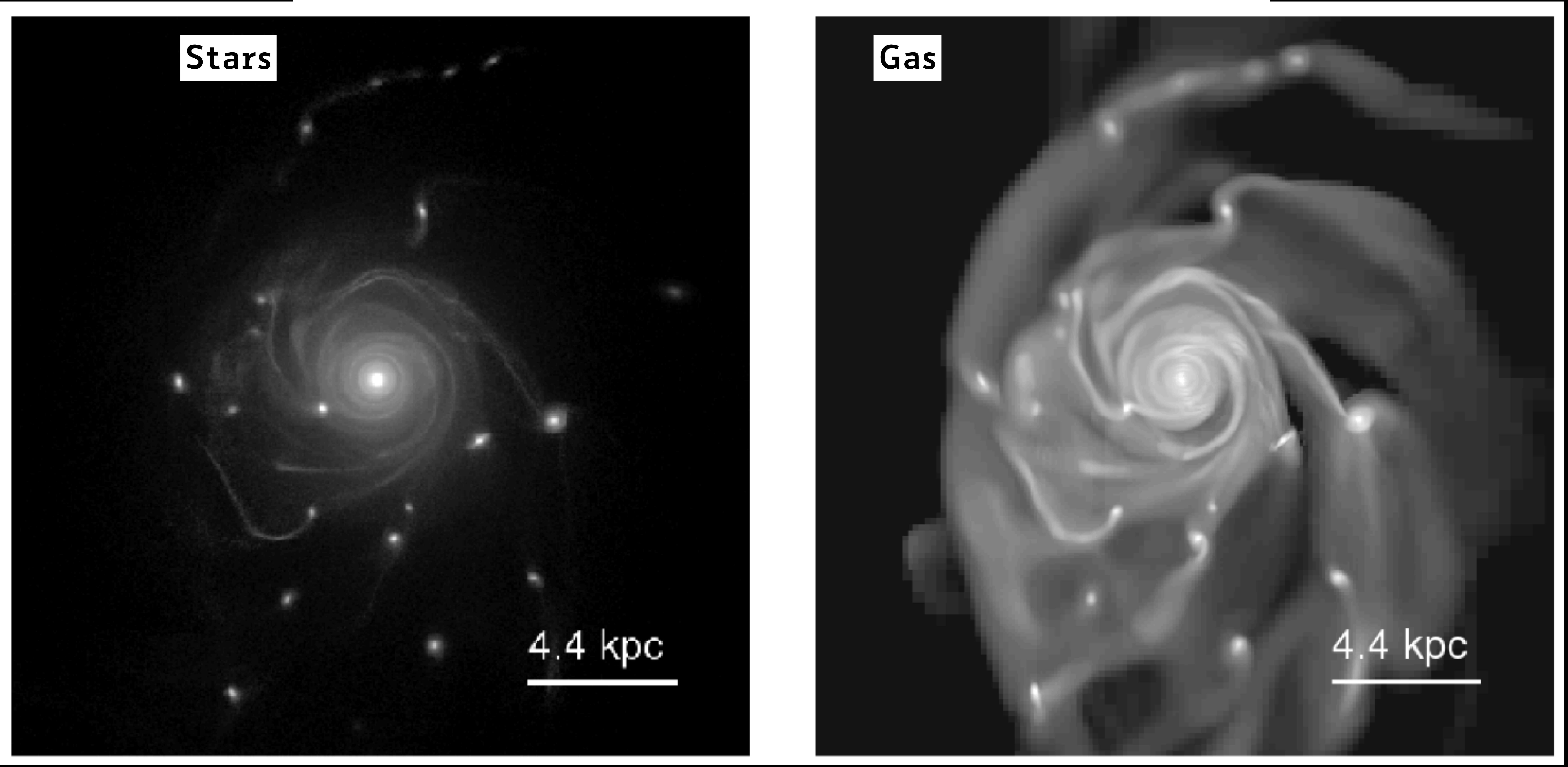


Looking for local gravitational instability in rotating fluids

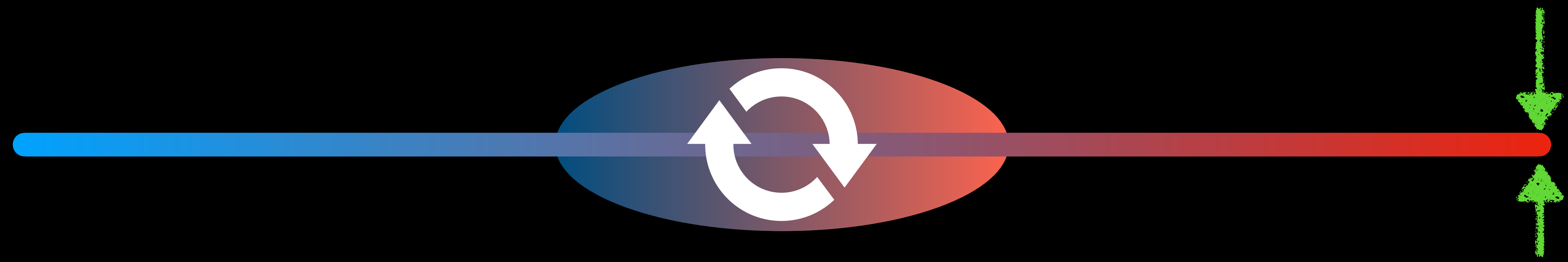


Gravitational instability -> gas clumps -> star cluster formation

Simulated disc galaxy at z=2.7 (Agertz et al. 2009)



Local gravitational instability in razor-thin gaseous discs (2D)

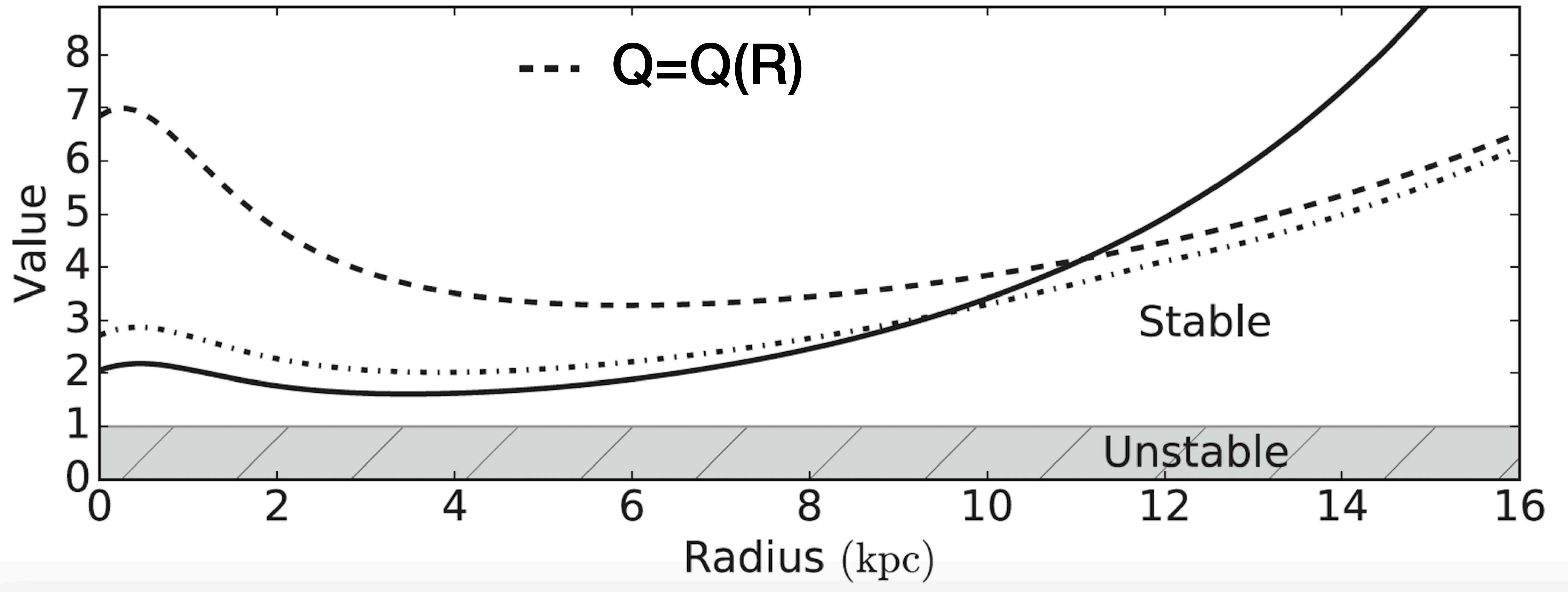


Geometrically thin

$$Q = \frac{\sigma\,\kappa}{\pi G \Sigma} < 1 \text{ for instability (Toomre 1964)}$$

- $\sigma(R)$ = velocity dispersion (pressure)
- $\kappa(R)$ = epicycle frequency (rotation)
- $\Sigma(R)$ = gas surface density (gravity)

$Q = Q(R)$

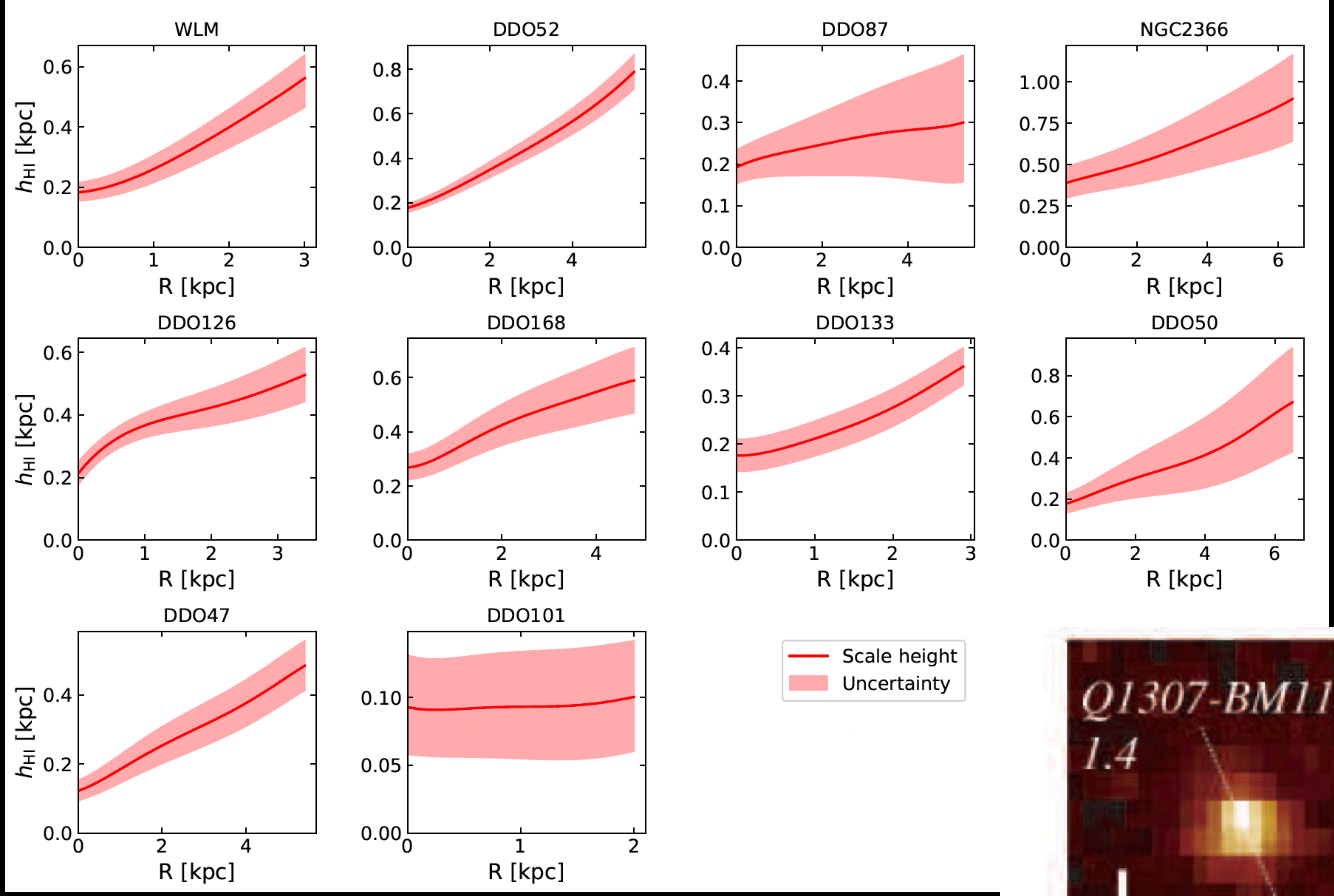


Cimatti, Fraternali, Nipoti (2019)

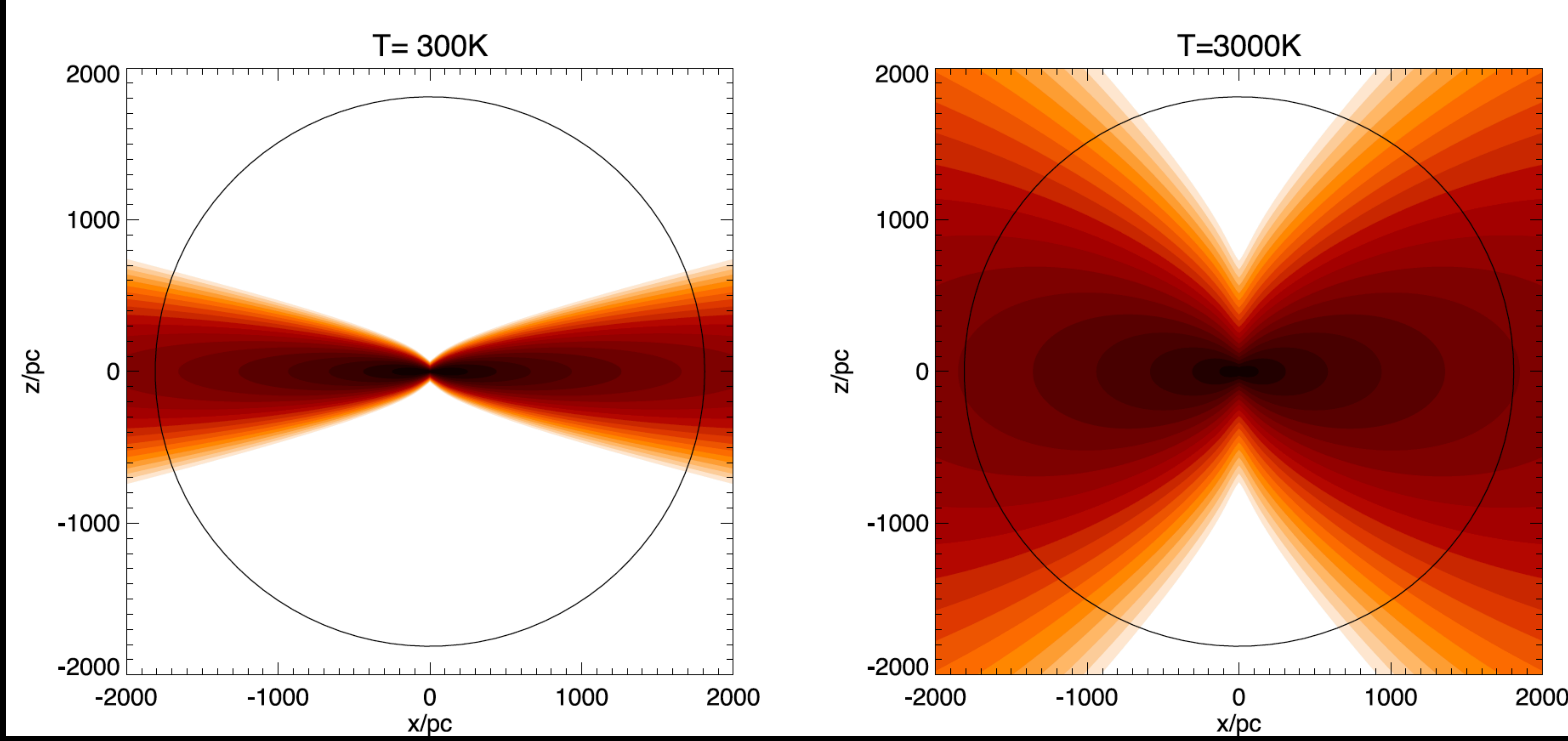
see also Lin & Shu (1964), Hunter (1972), Jog & Solomon (1984), Romeo (1992), Elmegreen (1995)

Not all galactic gaseous discs are thin

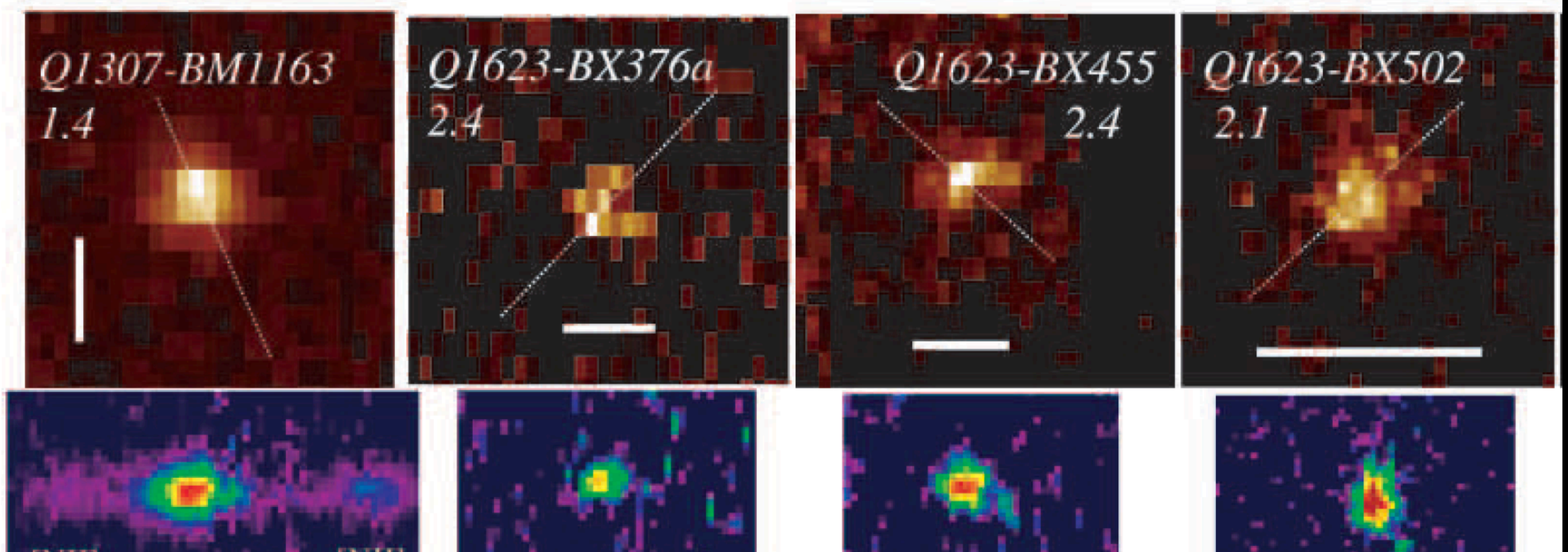
Nearby dwarfs (Bacchini et al. 2020)



Dwarf protogalaxies (Nipoti & Binney 2015)



High-redshift disc galaxies (Forster-Schreiber et al 2006)



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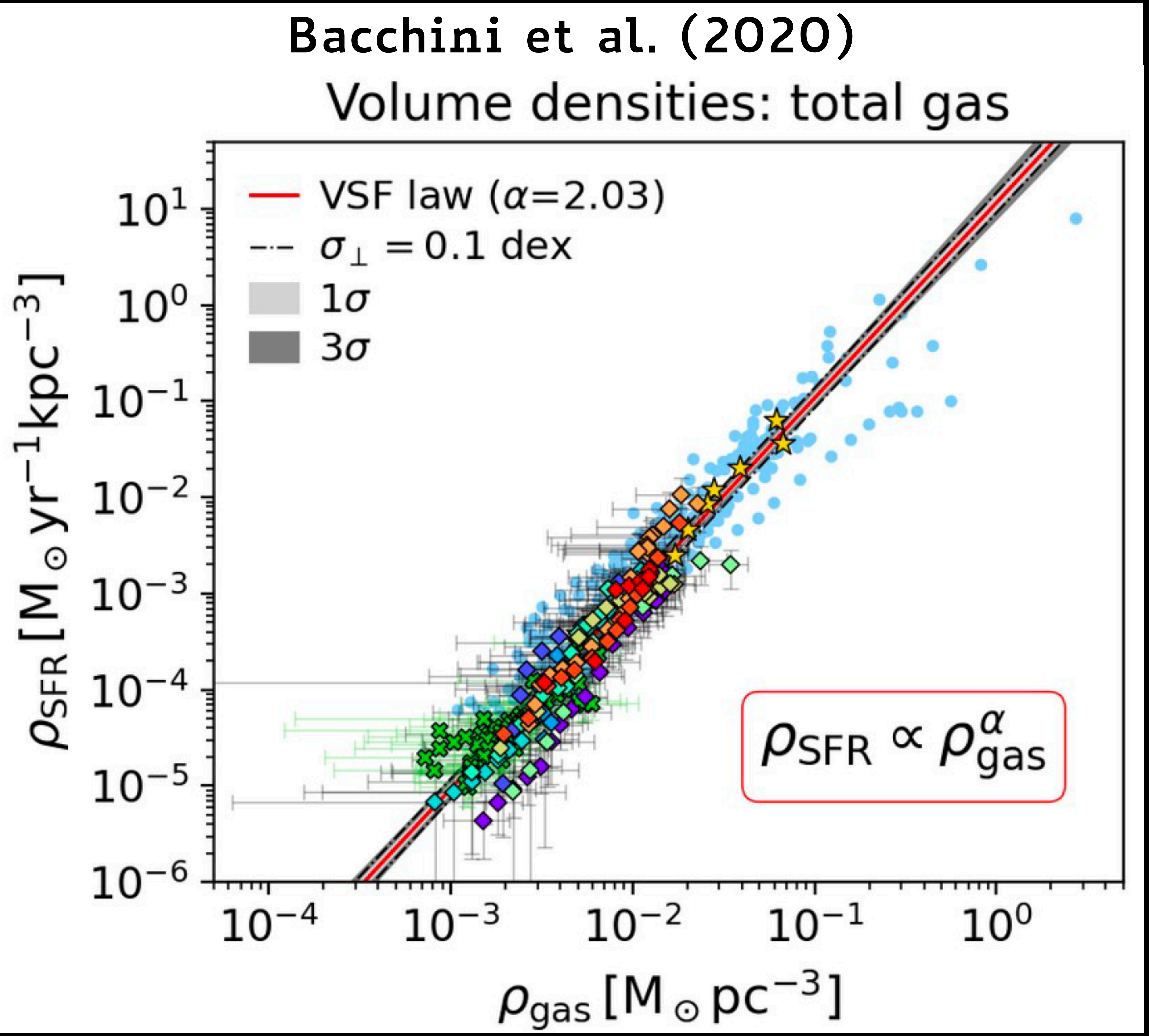
Volumetric star-formation laws in galaxies



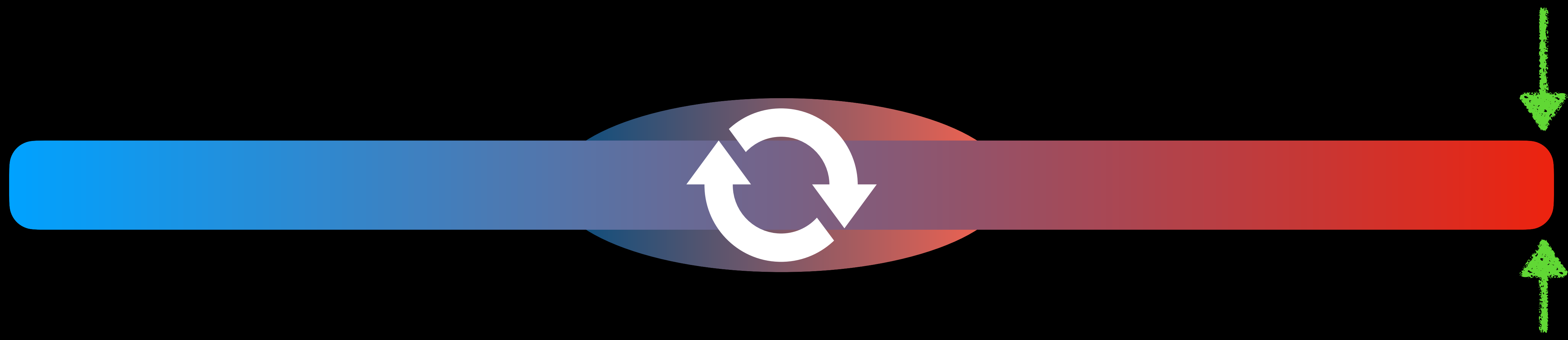
Local gravitational instability in geometrically thick gaseous discs: 2D+correction

$Q < Q_{\rm crit} < 1$ for instability (e.g. Vandervoort 1970)

- (Stabilizing) reduction factor $\mathscr{F}(h_z, k) < 1$ in the dispersion relation
- h_z = disc thickness
- k = wave number

(see also Toomre 1964; Shu 1968; Yue 1982, Bertin & Amorisco 2010; Wang et al. 2010; Elmegreen 2011; Griv & Gedalin 2012; Romeo & Falstad 2013; Behrendt et al. 2015)

Local gravitational instability in vertically stratified gaseous discs: 3D analysis of specific models



Assuming uniform rotation and polytropic equation of state $p \propto \rho^{\gamma'}$:

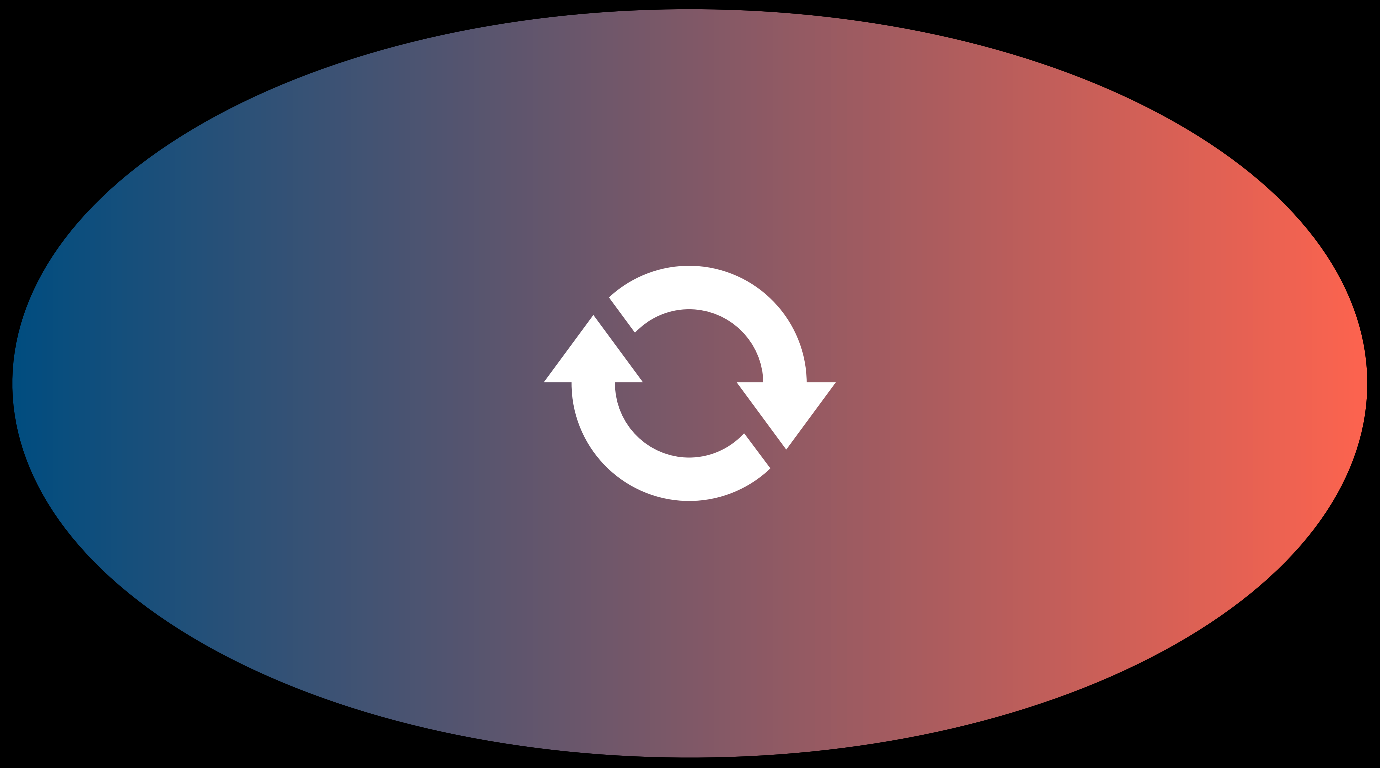
$$\frac{\pi G \bar{\rho}}{4\Omega^2} \gtrsim 1$$ for instability (Goldreich & Lynden-Bell 1965)

- $\bar{\rho}(R) =$ vertically-averaged mean gas density
- $\Omega =$ angular velocity

(see also Safronov 1960, Chandrasekhar 1961, Genkin & Safronov 1975, Bertin & Casertano 1982, Mamatsashvili & Rice 2010, Meidt 2022)

3D local gravitational instability in general stratified rotating fluids

(Nipoti 2023)



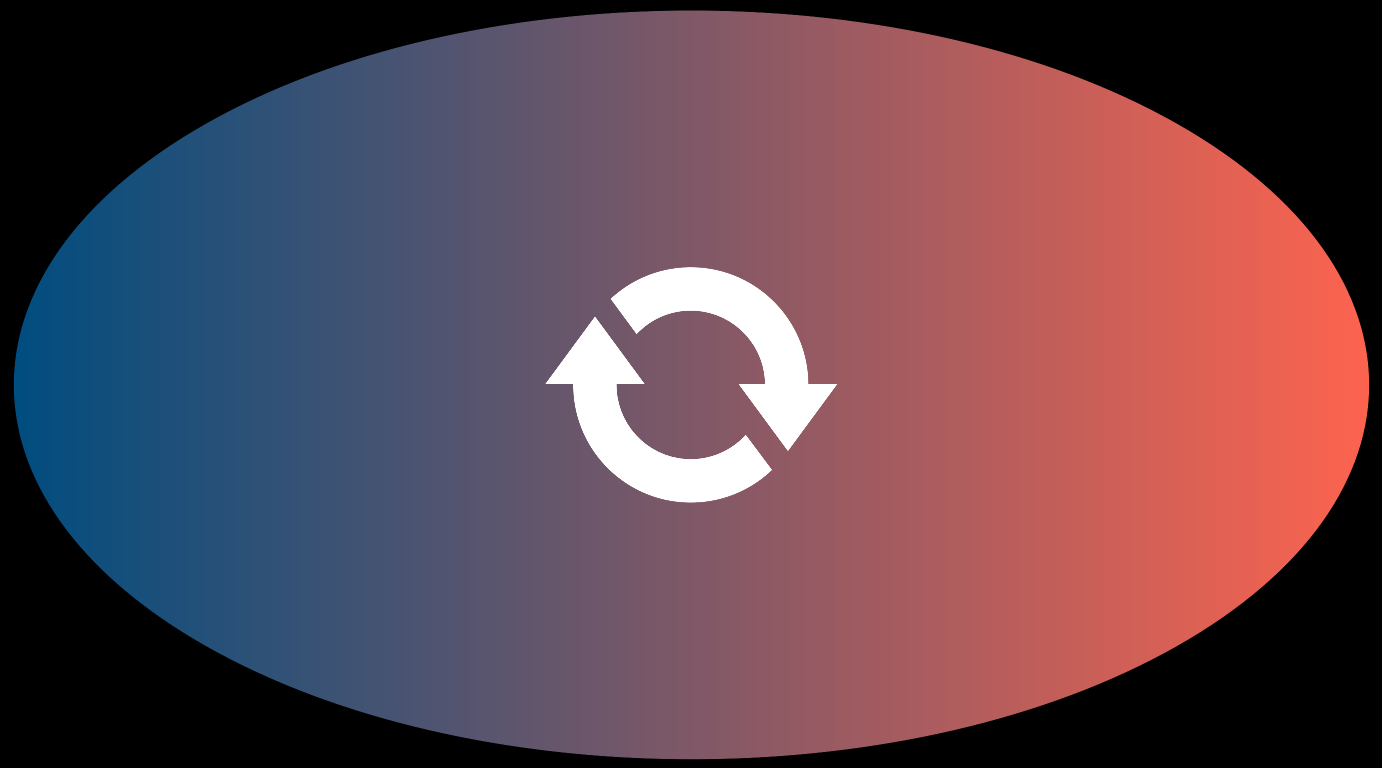
Ingredients:

- Inviscid, unmagnetized, axisymmetric hydro equations
- Poisson equation
- $\rho(R, z) =$ volume gas density
- $\Omega(R, z) =$ angular velocity
- $p(R, z) =$ gas pressure
- $\Phi(R, z) = \Phi_{\rm gas} + \Phi_{\rm ext} =$ gravitational potential
- Ideal gas equation of state

Dispersion relation for axisymmetric perturbations

(Nipoti 2023)

$$\omega^4 + \left(4\pi G\rho - \kappa^2 - \nu^2 - c_s^2 k^2\right)\omega^2 + \mathcal{N}^2 c_s^2 k^2 + c_s^2 k_z \left(k_z \kappa^2 - k_R R \frac{\partial \Omega^2}{\partial z}\right) - 4\pi G\rho \frac{k_z}{k^2}\left(k_z \kappa^2 - k_R R \frac{\partial \Omega^2}{\partial z}\right) + \kappa^2 \nu^2 = 0,$$

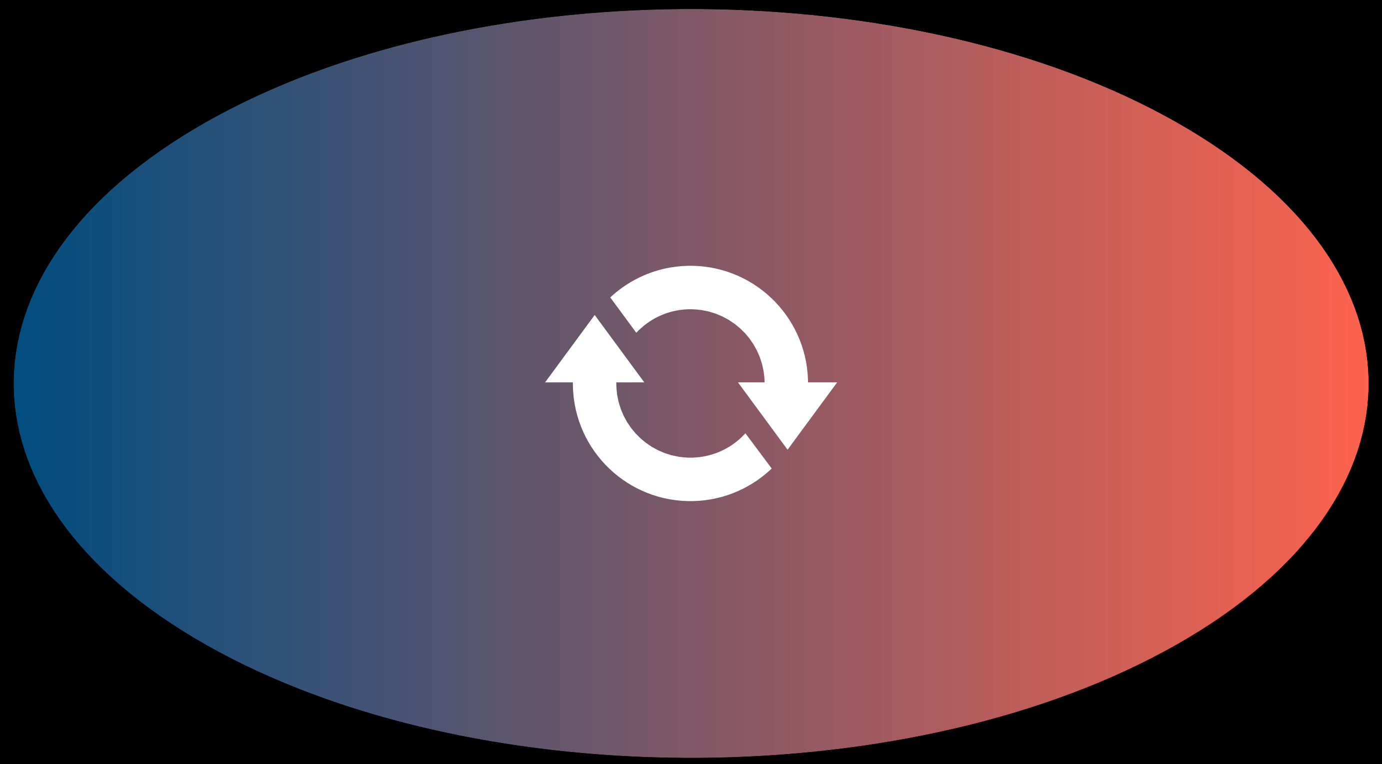


$$\nu^2 \equiv \frac{\rho'_z p'_z}{\rho^2}$$

$$\mathcal{N}^2 \equiv -\frac{1}{\gamma\rho}\left[\frac{k_z^2}{k^2}\sigma'_R p'_R + \frac{k_R^2}{k^2}\sigma'_z p'_z - \frac{k_R k_z}{k^2}\left(\sigma'_R p'_z + \sigma'_z p'_R\right)\right]$$

3D criterion for general baroclinic or barotropic distributions

(Nipoti 2023)



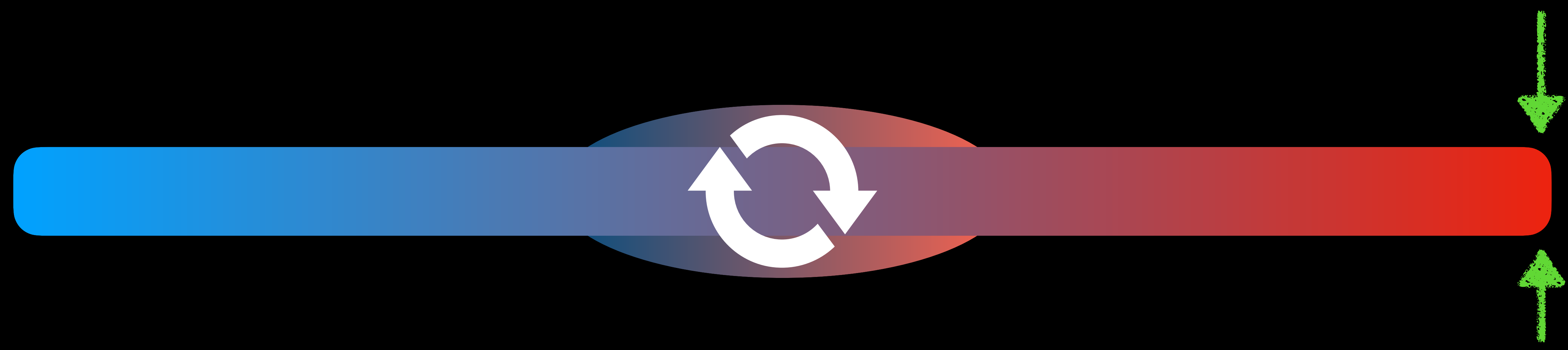
$$4\pi G\rho N_z^2 > \nu^2(\nu^2 - N_z^2) + \left(\kappa^2/2\right)^2$$

(sufficient for instability)

- $\rho(R, z) =$ volume gas density (gravity)
- $N_z^2(R, z) =$ Brunt-Vaisala frequency (buoyancy)
- $\nu^2(R, z) \equiv \frac{1}{\rho^2}\frac{\partial\rho}{\partial z}\frac{\partial p}{\partial z} =$ "vertical gradient" frequency (stratification)
- $\kappa(R, z) =$ epicycle frequency (rotation) when $\Omega = \Omega(R, z)$

Local gravitational instability in vertically stratified gaseous discs: analytic 3D dispersion relation

(Nipoti 2023)



- Unperturbed disc: $\rho = \rho(z)$, $\quad p = p(z)$, $\quad \Omega = \Omega(R)$, $\quad \Phi = \Phi(R, z)$
- Axisymmetric perturbations: ω = frequency, $\boldsymbol{k} = (k_R, k_z)$ = wavevector

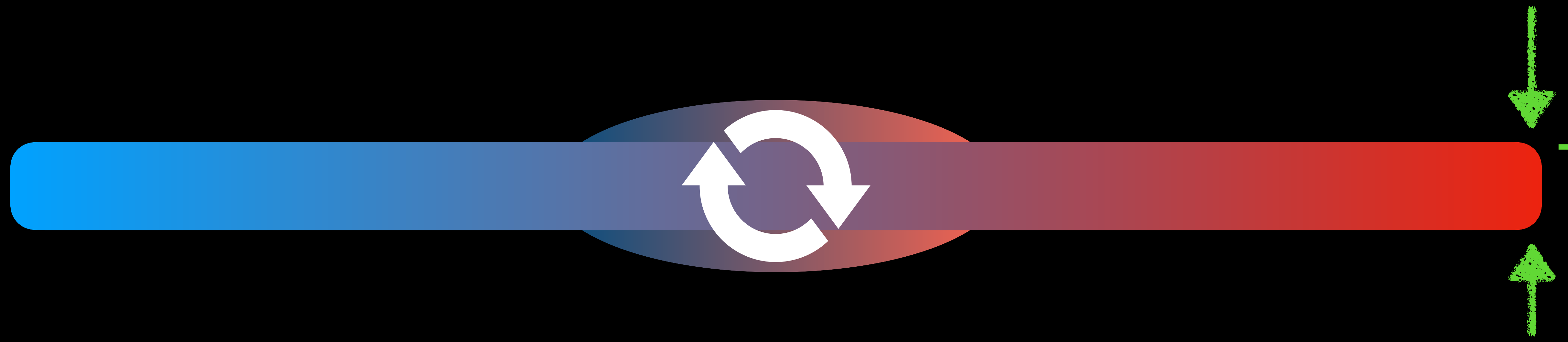
$\Longrightarrow$ Biquadratic dispersion relation:

$$\boxed{\omega^4 + B\omega^2 + C = 0}$$

$$B = B(R, z, k_R, k_z)$$

$$C = C(R, z, k_R, k_z)$$

3D instability criterion for vertically stratified discs: $Q_{3D} = Q_{3D}(R, z)$



$$Q_{3D} \equiv \frac{\sqrt{\kappa^2 + \nu^2} + \sigma h_z^{-1}}{\sqrt{4\pi G \rho}} < 1$$

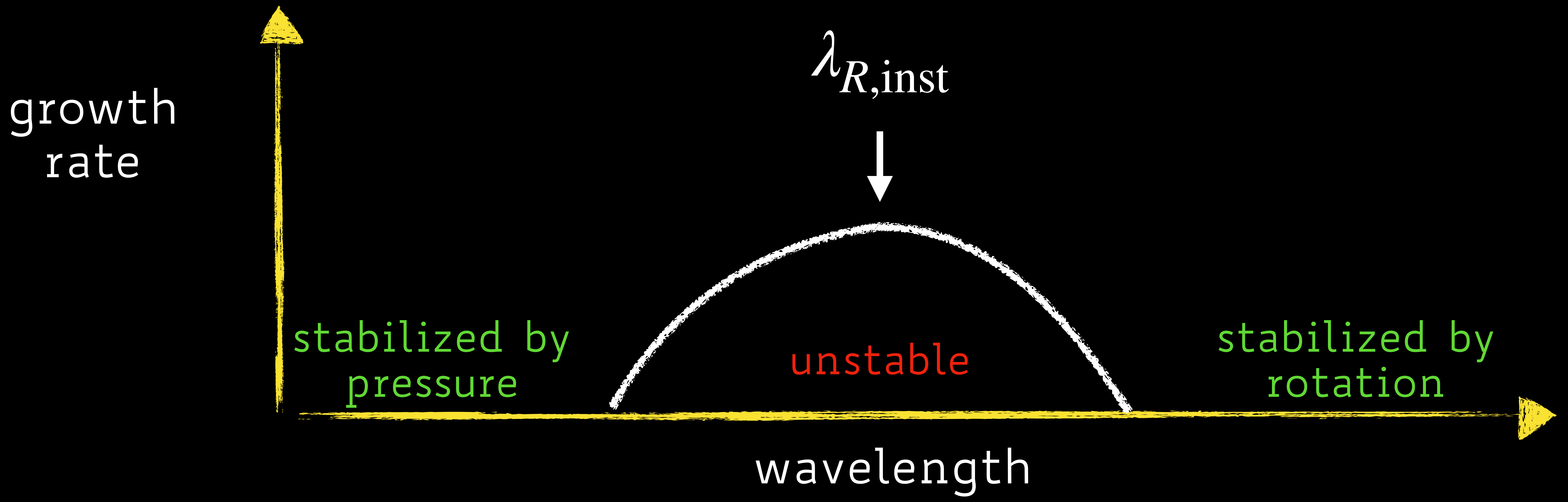
(sufficient for instability)

(Nipoti 2023)

- h_z = disc thickness ($h_z \approx h_{70\%}$ contains 70% of gas mass)
- ρ = volume gas density (gravity)
- κ = epicycle frequency (rotation)
- $\nu^2 \equiv \frac{1}{\rho^2}\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial z} =$ "vertical gradient" frequency (stratification)

Where $Q_{\rm 3D} < 1$ the fastest growing perturbations have radial wavelength $\lambda_{R,\rm inst} \approx 2\pi h_z$

(Nipoti 2023)



3D stability criterion for vertically stratified discs (at any R and z)

(Nipoti 2023)

$$4\pi G\rho N_z^2 < (\nu^2 - N_z^2)(N_z^2 - \kappa^2)$$ (sufficient for stability)

- $\rho(z) =$ volume gas density (gravity)
- $N_z^2(z) =$ Brunt-Vaisala frequency (buoyancy)
- $\nu^2(z) \equiv \frac{1}{\rho^2}\frac{\partial \rho}{\partial z}\frac{\partial p}{\partial z} =$ "vertical gradient" frequency (stratification)
- $\kappa(R) =$ epicycle frequency (rotation)

A case study: self-gravitating disc in vertical isothermal equilibrium

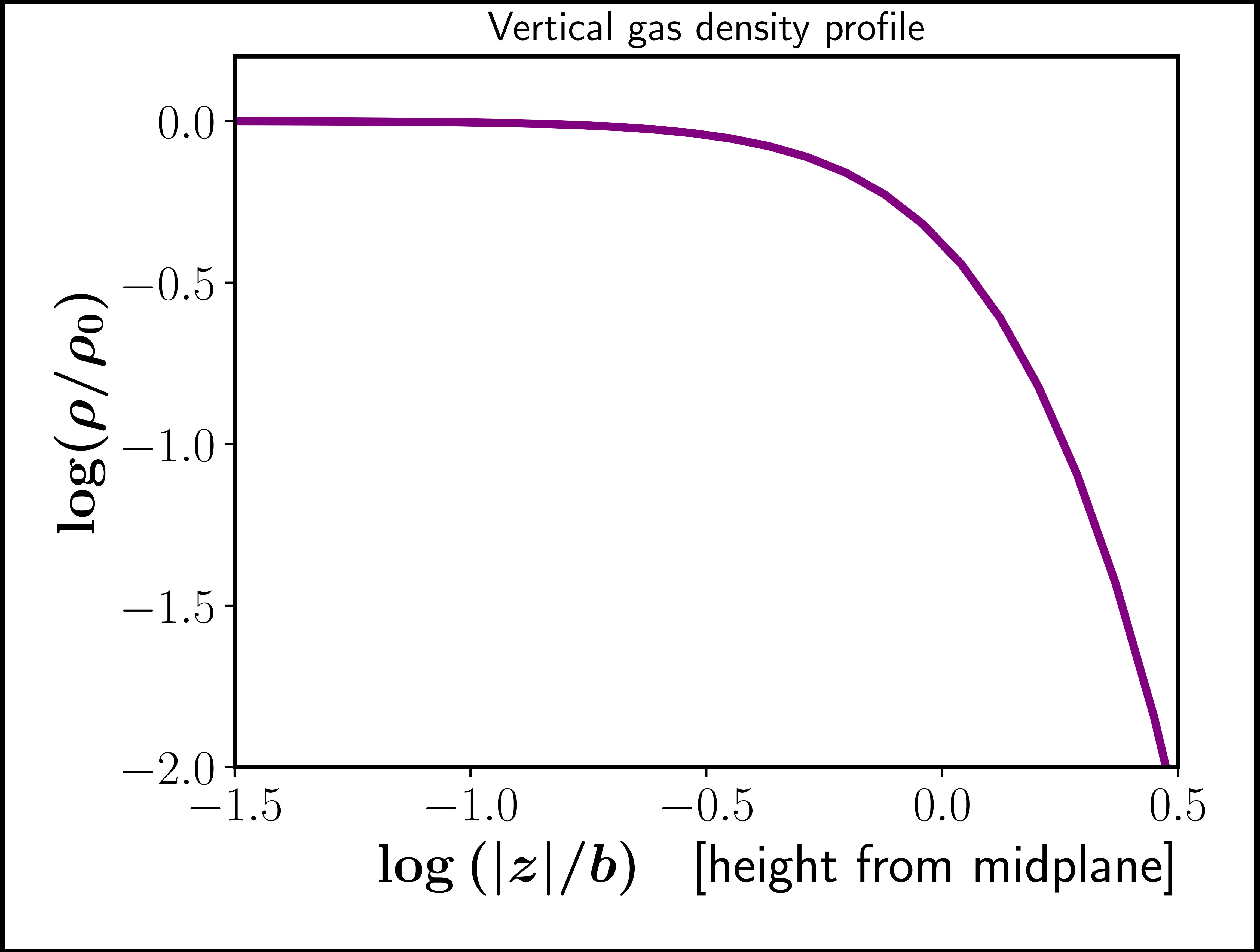
- Isothermal slab (Spitzer 1942):

$$\rho(z) = \rho_0 \, \mathrm{sech}^2\left(\frac{z}{b}\right)$$

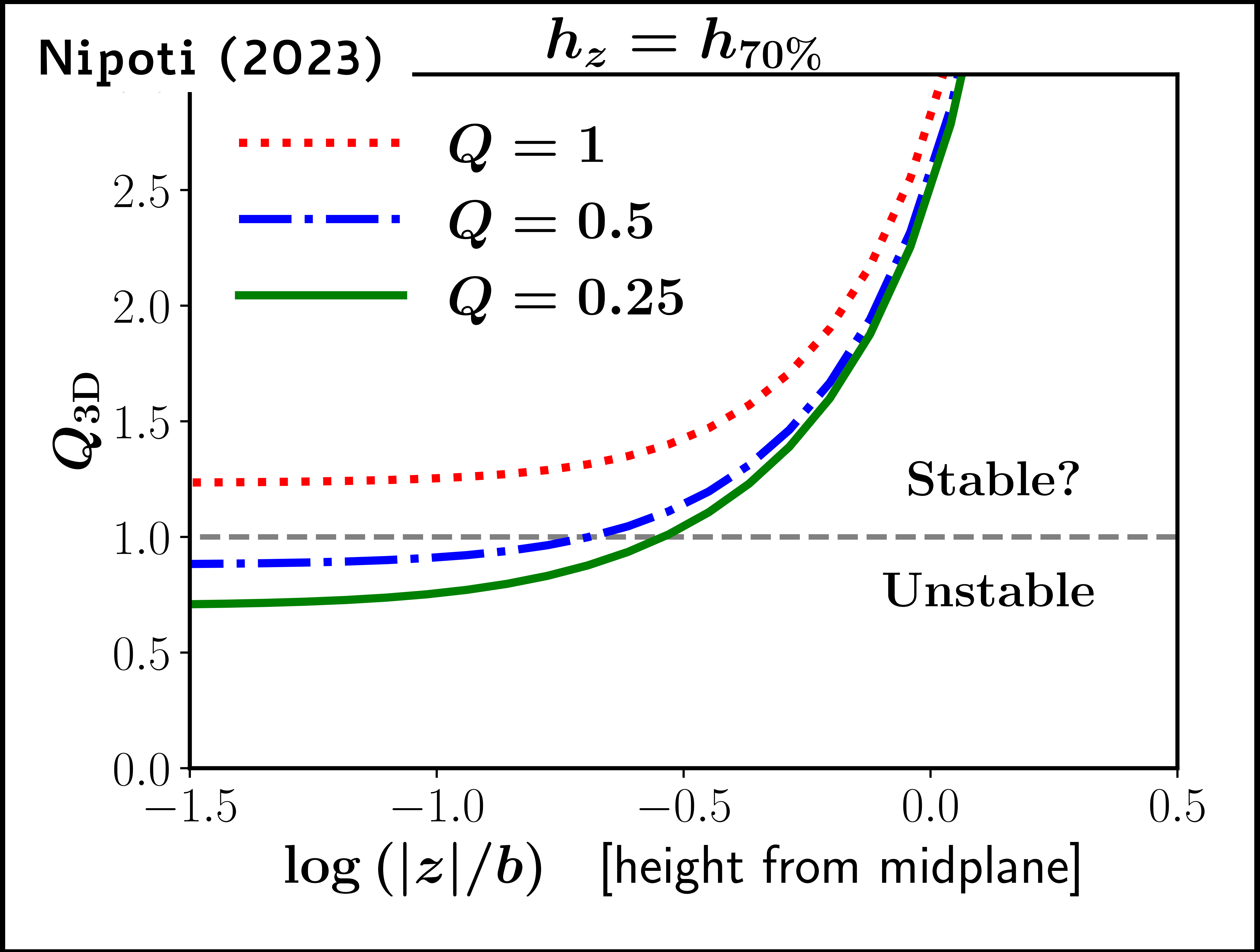
- Scale height: $b \equiv \dfrac{\sigma}{\sqrt{2\pi G \rho_0}}$

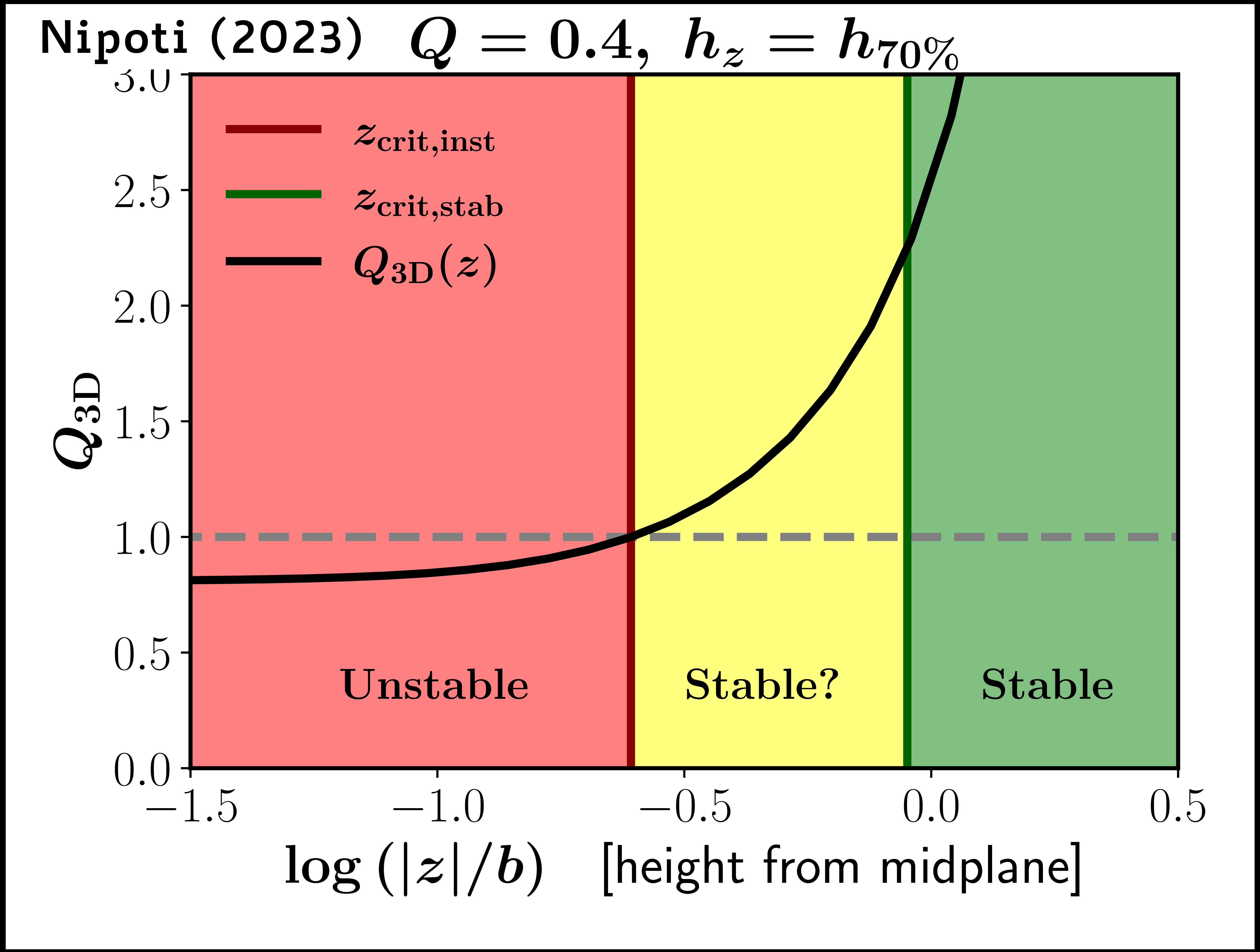
- Disc thickness $h_z \propto b \propto \dfrac{\sigma}{\rho_0^{1/2}}$

- $Q_{3D} = Q_{3D}(R, z)$



Q_{3D} vs. height from disc midplane at given R (isothermal discs)

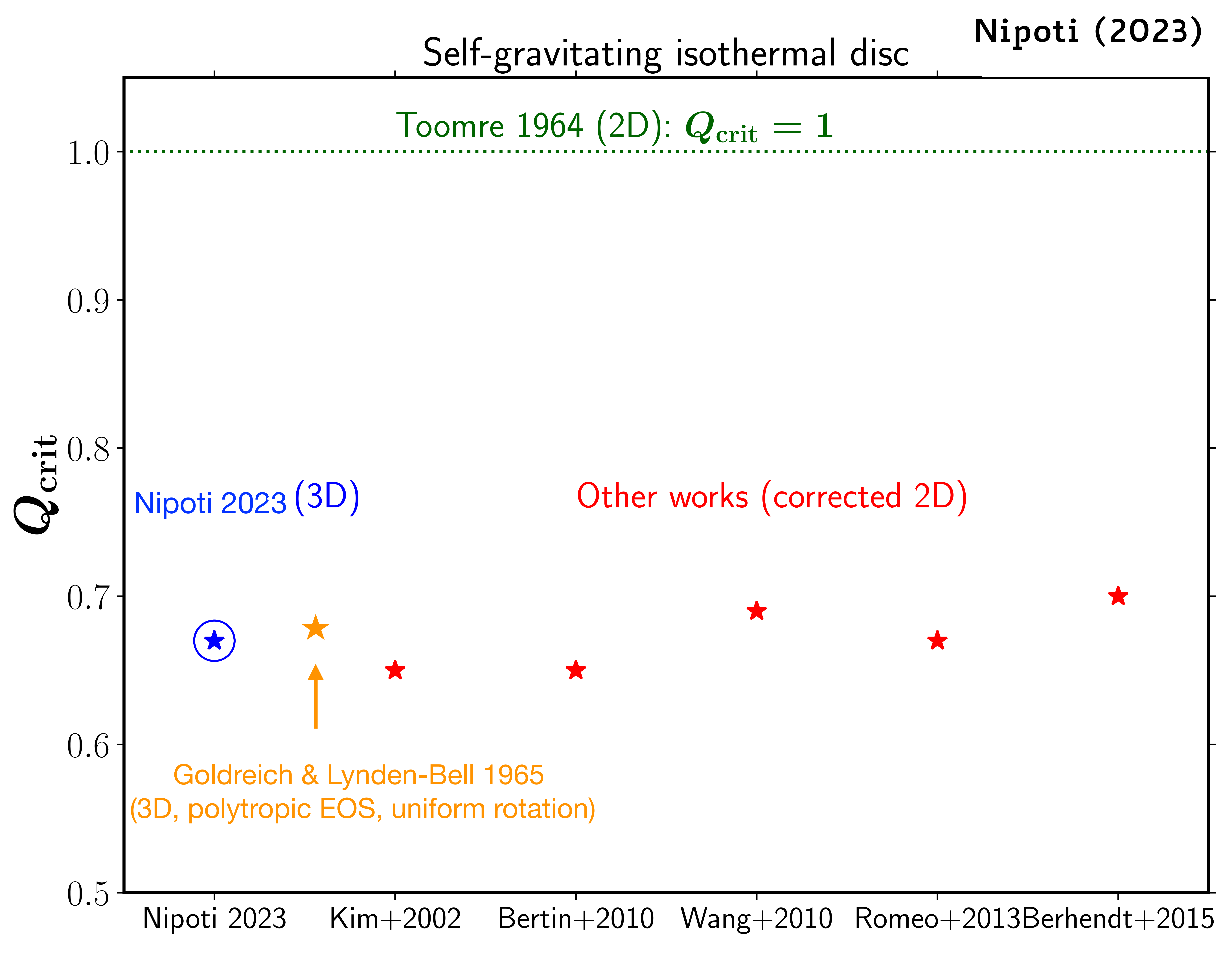




- Self-gravitating stratified disc in vertical isothermal equilibrium
- Higher Q $\Longleftrightarrow$ more rotation support (Q = Toomre's 2D parameter)

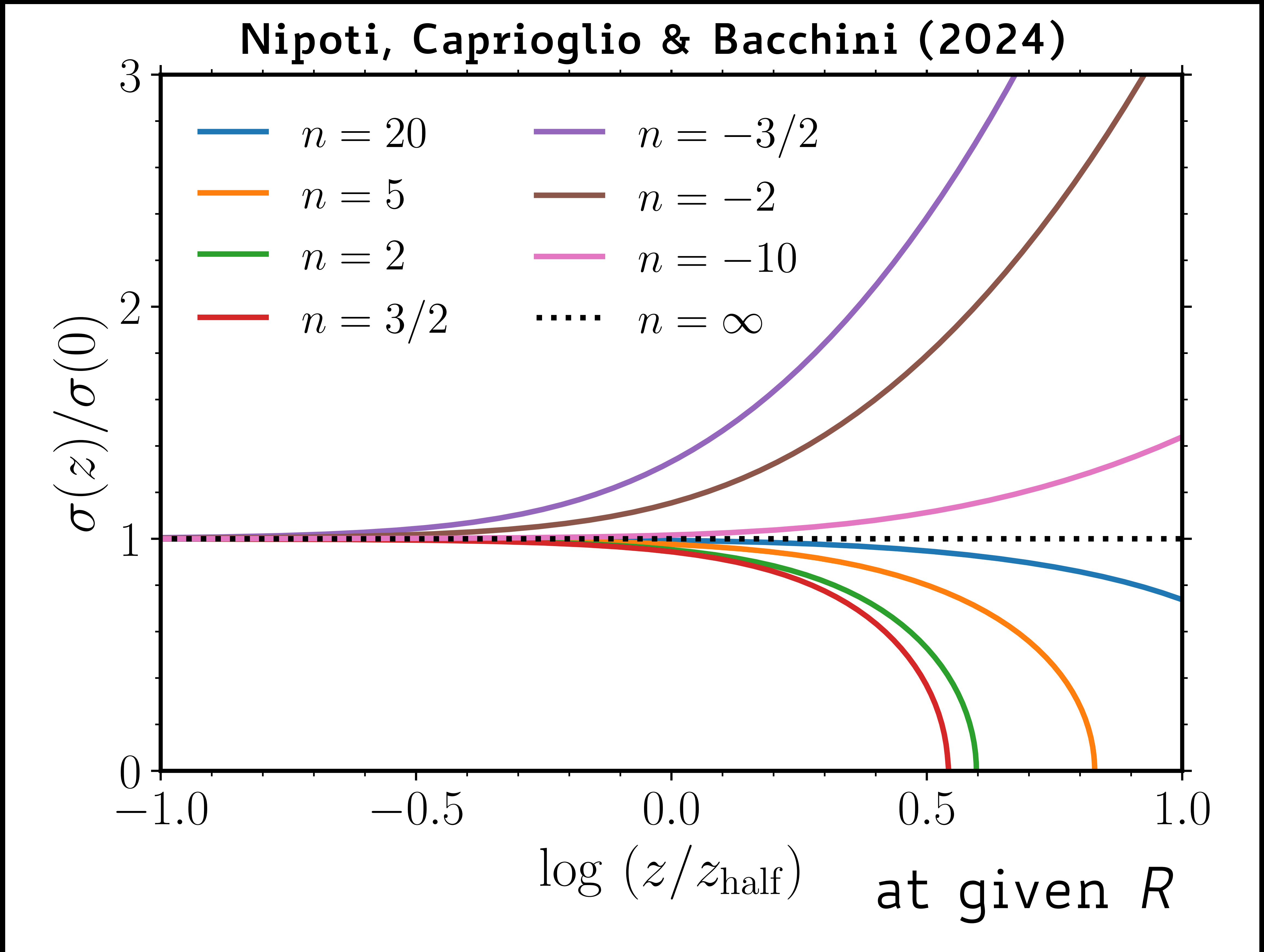
Comparison with previous works

$Q(R) < Q_{\rm crit}$
for instability

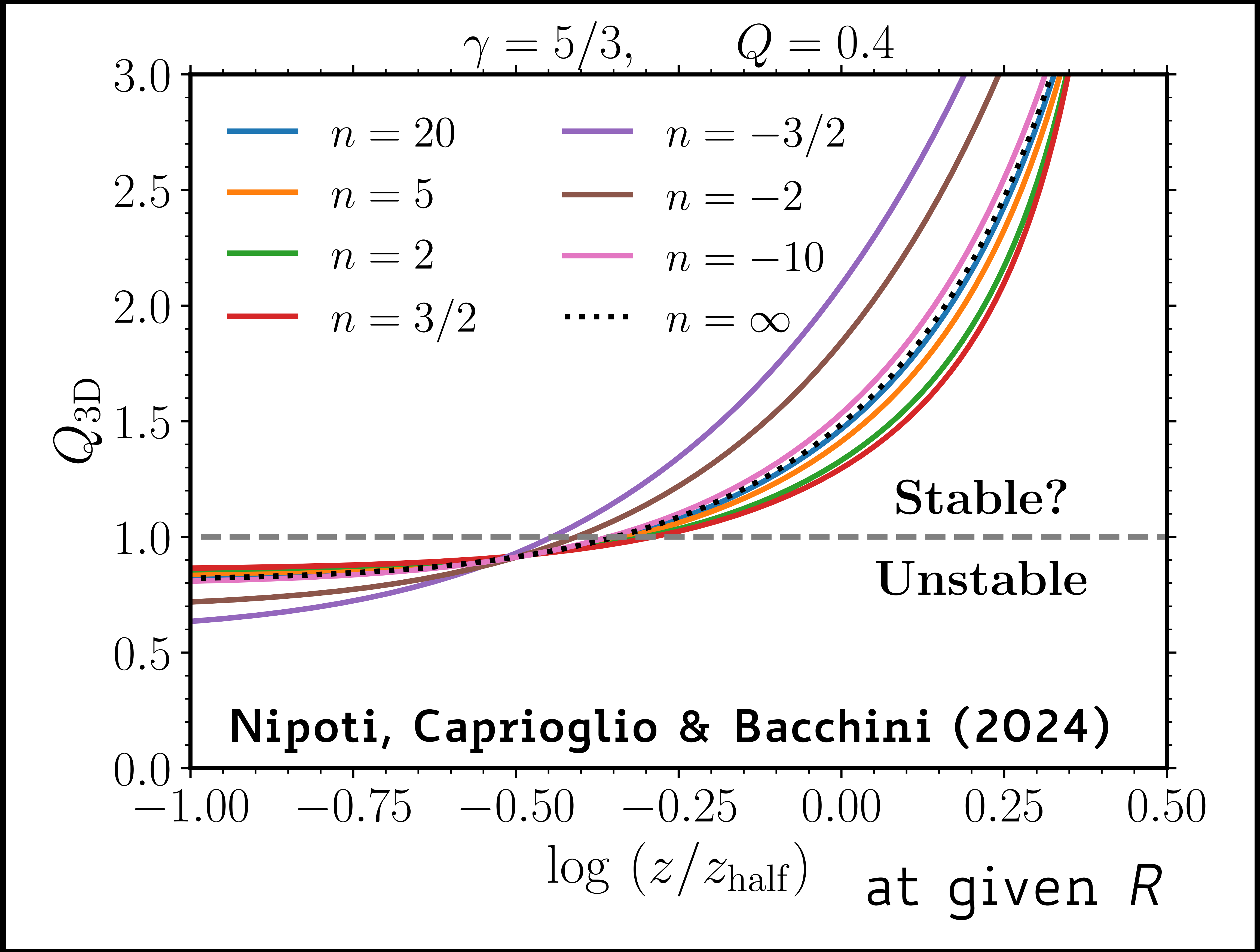


(see also Safronov 1960, Chandrasekhar 1961, Shu 1968, Vandervoort 1970; Genkin & Safronov 1975, Bertin & Casertano 1982, Yue 1982, Mamatsashvili & Rice 2010, Elmegreen 2011, Griv & Gedalin 2012, Meidt 2022)

Self-gravitating thick discs with vertical velocity-dispersion gradients



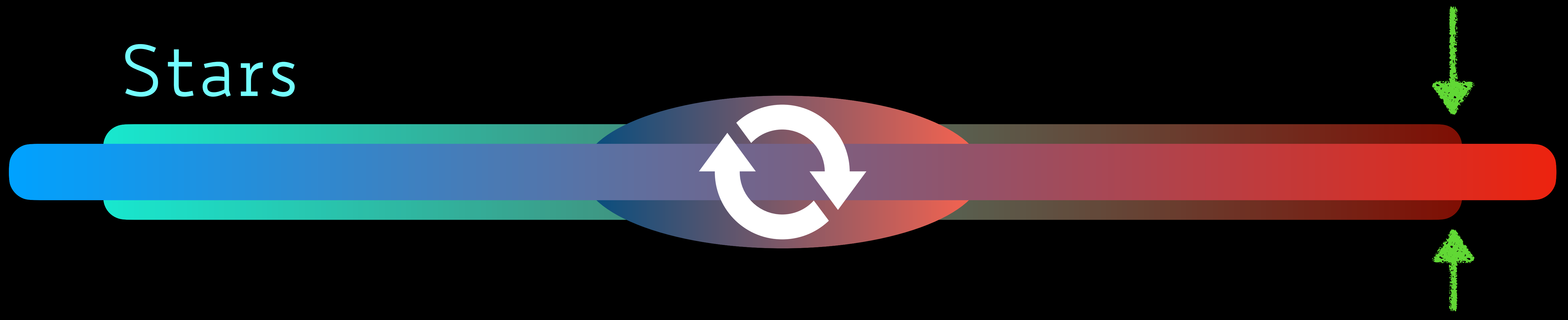
Vertical velocity-dispersion profiles



Vertical $Q_{\rm 3D}$ profiles

Polytropic vertical distributions: n=polytropic index

3D instability of two-component vertically stratified discs: gas+stars



$Q_{\rm 3D,gas} < 1$ or $Q_{\rm 3D,\star} < 1$

INSTABILITY

$Q_{\rm 3D,gas} > 1$ and $Q_{\rm 3D,\star} > 1$

$F_{\rm gas+\star}(k_R) > 1$

INSTABILITY (back-reaction)

$k_R =$ perturbation wavenumber

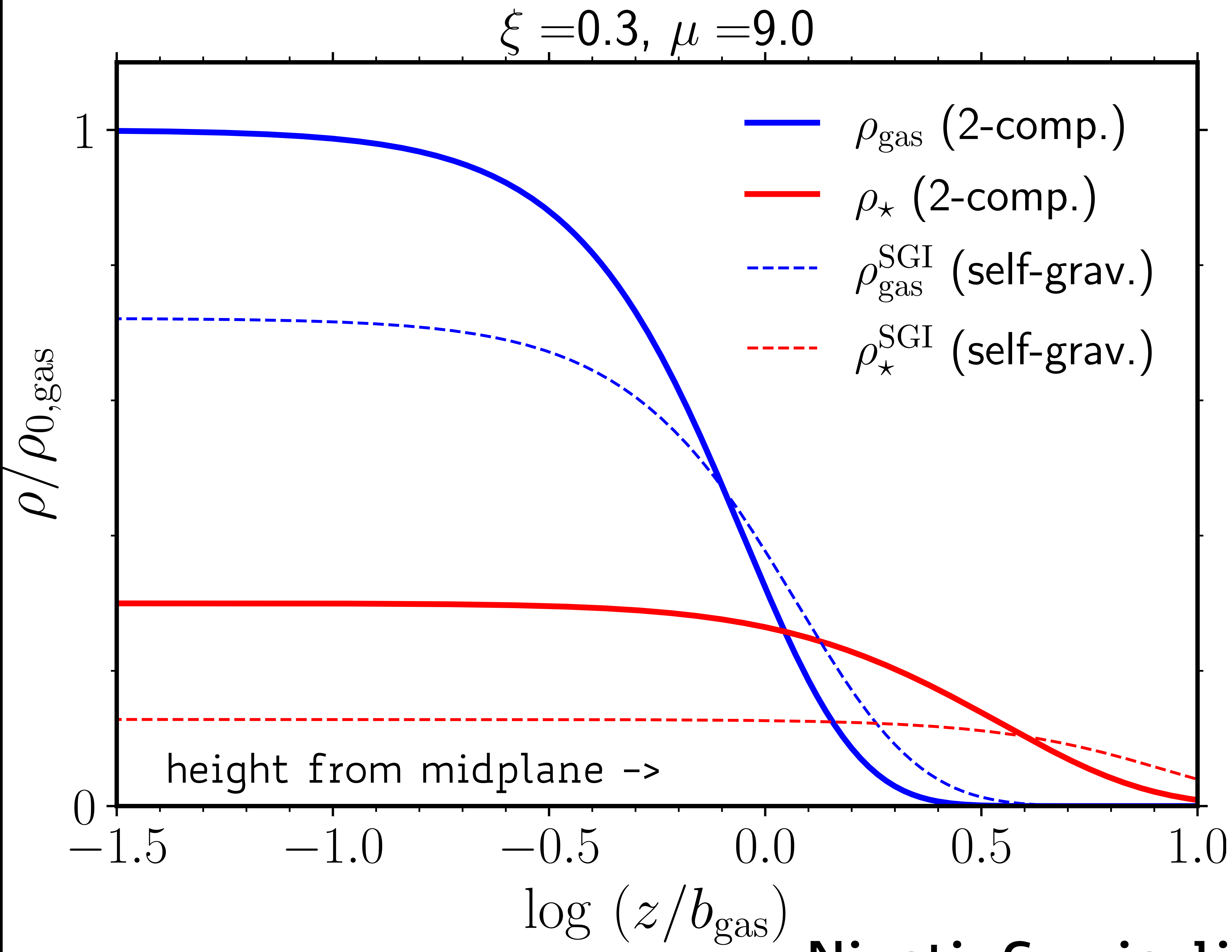
$$F_{\rm gas+\star}(k_R) = F_{\rm gas}(k_R) + F_\star(k_R)$$

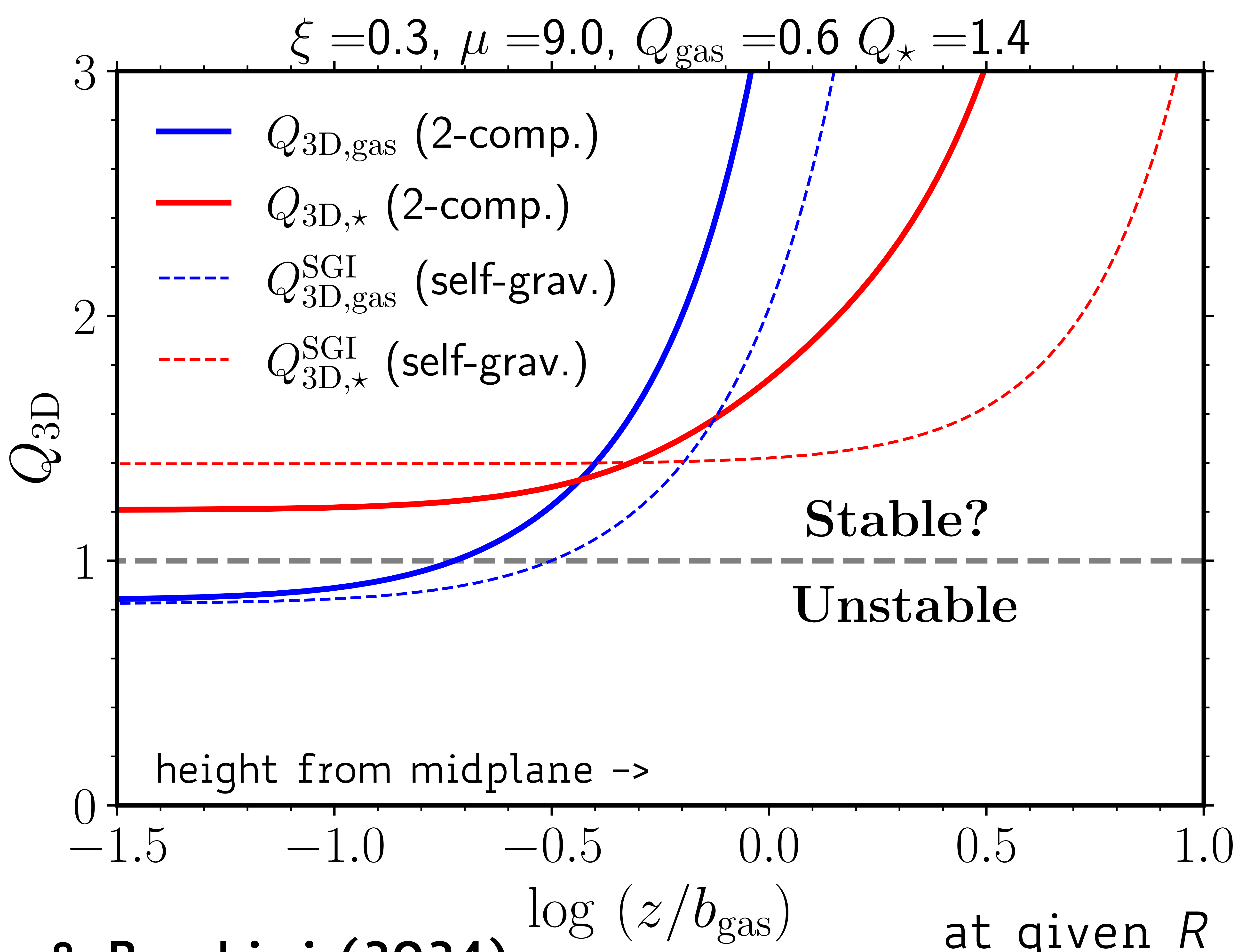
$$F_{\rm gas}(k_R) = \frac{2\tilde{k}_R^2}{(\gamma\tilde{k}_R^2 + Q_{\rm gas}^2\tilde{\Sigma}_{\rm gas}^2)(\tilde{k}_R^2 + \tilde{h}_{z,\rm gas}^{-2})}$$

$$F_\star(k_R) = \frac{2\xi\tilde{k}_R^2}{(\mu\gamma\tilde{k}_R^2 + Q_\star^2\tilde{\Sigma}_\star^2/\mu)(\tilde{k}_R^2 + \tilde{h}_{z,\star}^{-2})}$$

Nipoti, Caprioglio & Bacchini (2024)

3D instability of thick gas+stars isothermal discs: no back-reaction





at given R

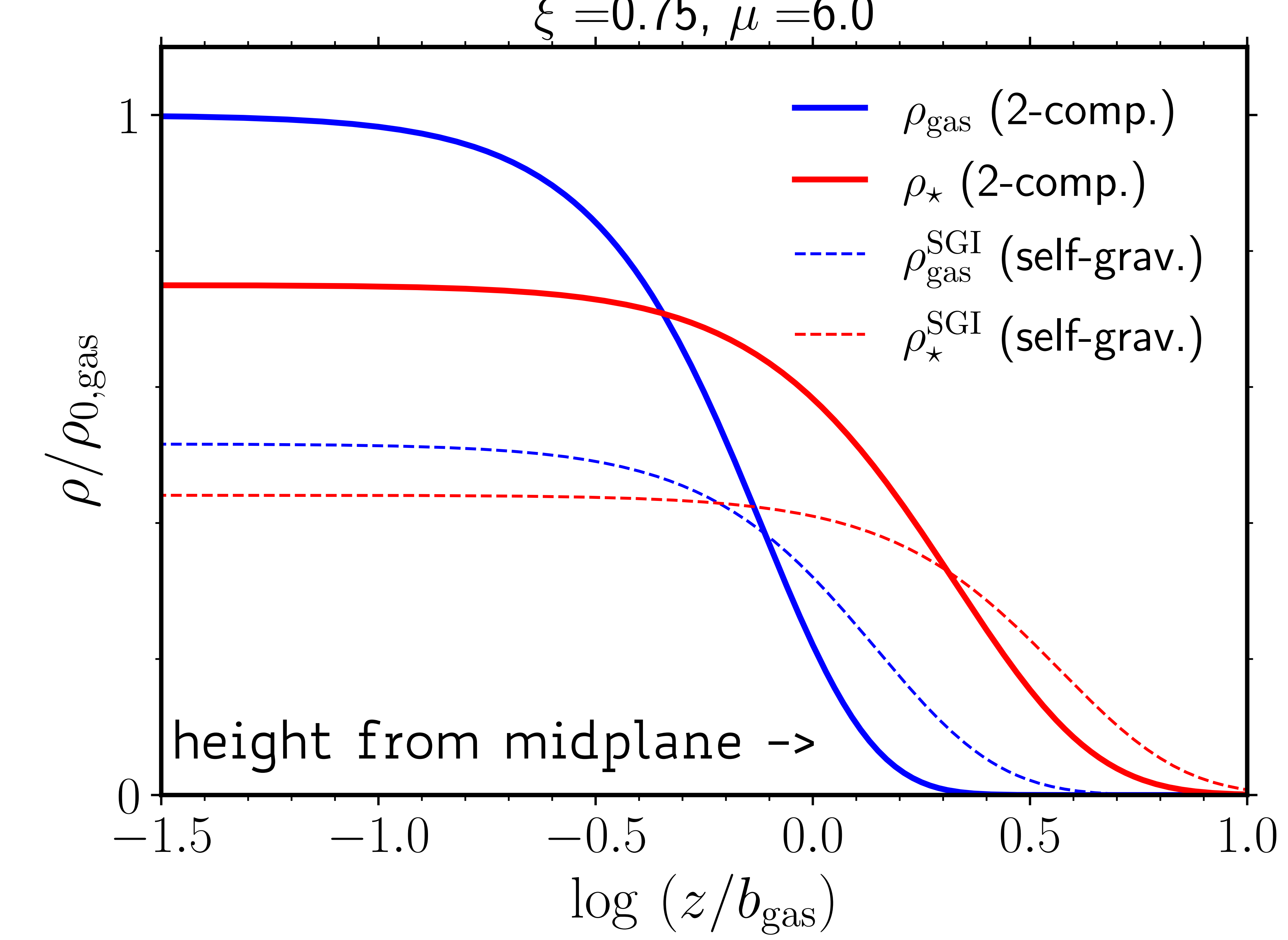
Nipoti, Caprioglio & Bacchini (2024)

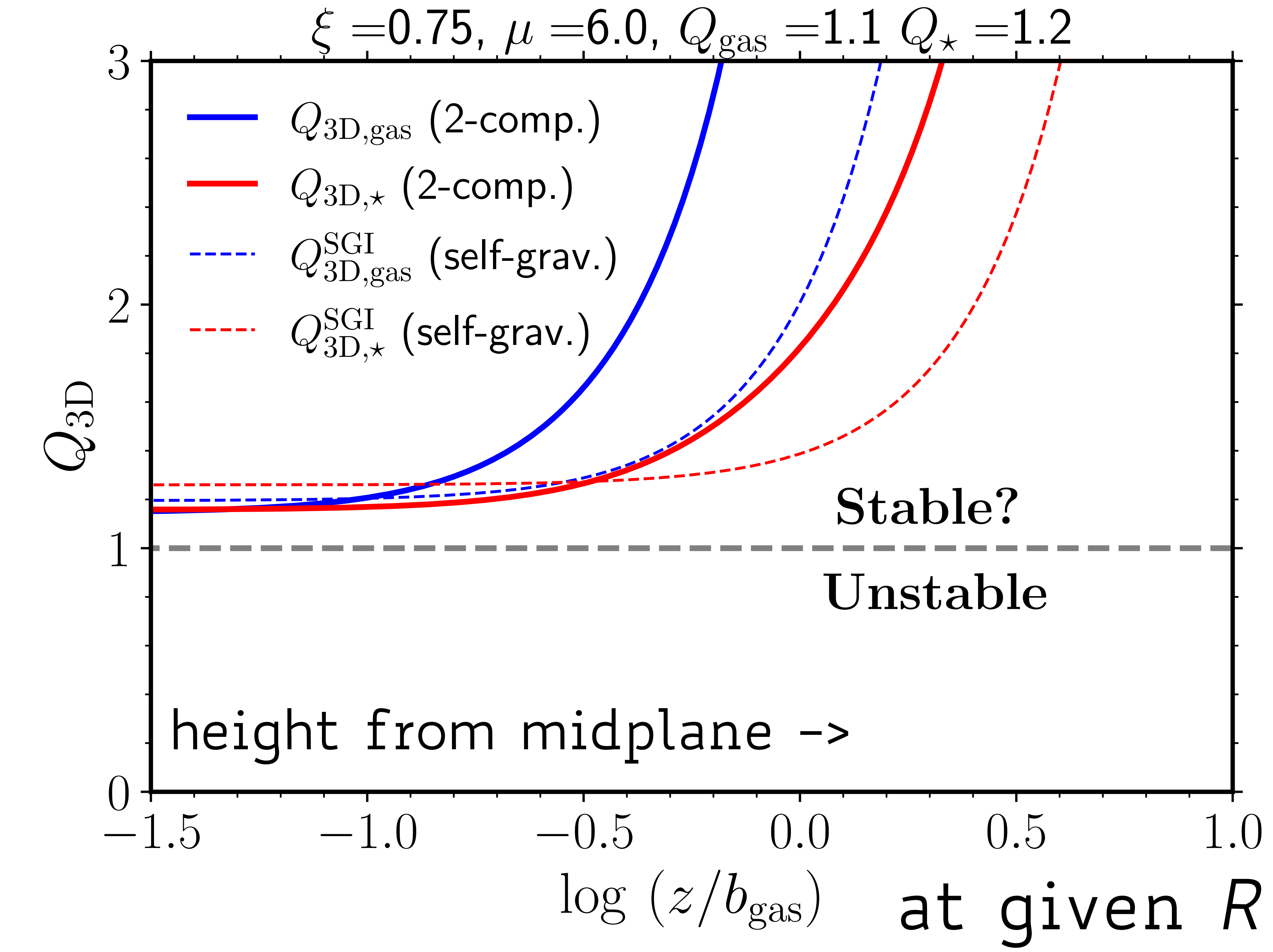
Vertical density profiles

(see also Bertin & Pegoraro 2022)

Vertical Q_{3D} profiles

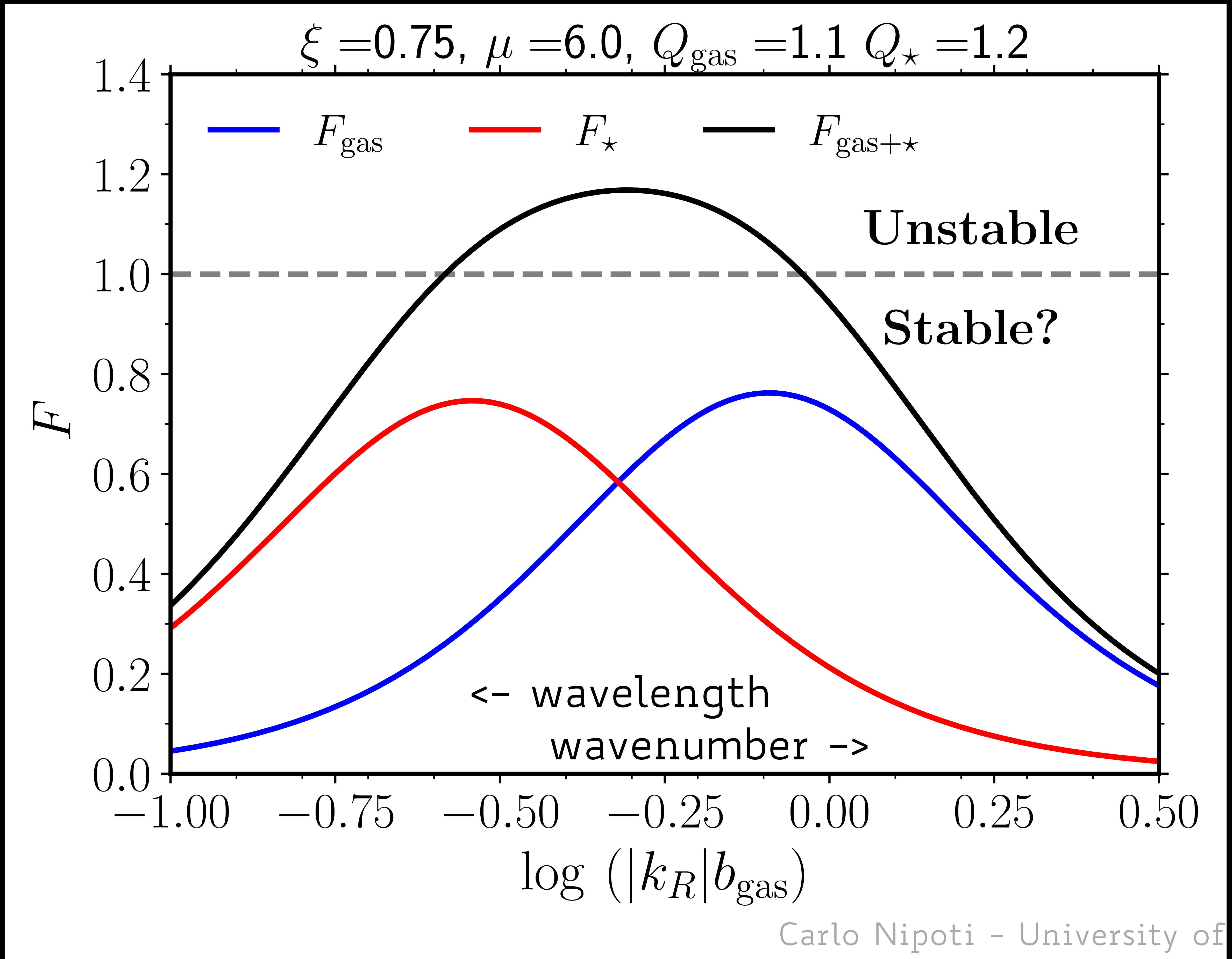
3D instability of thick gas+stars isothermal discs: with back-reaction



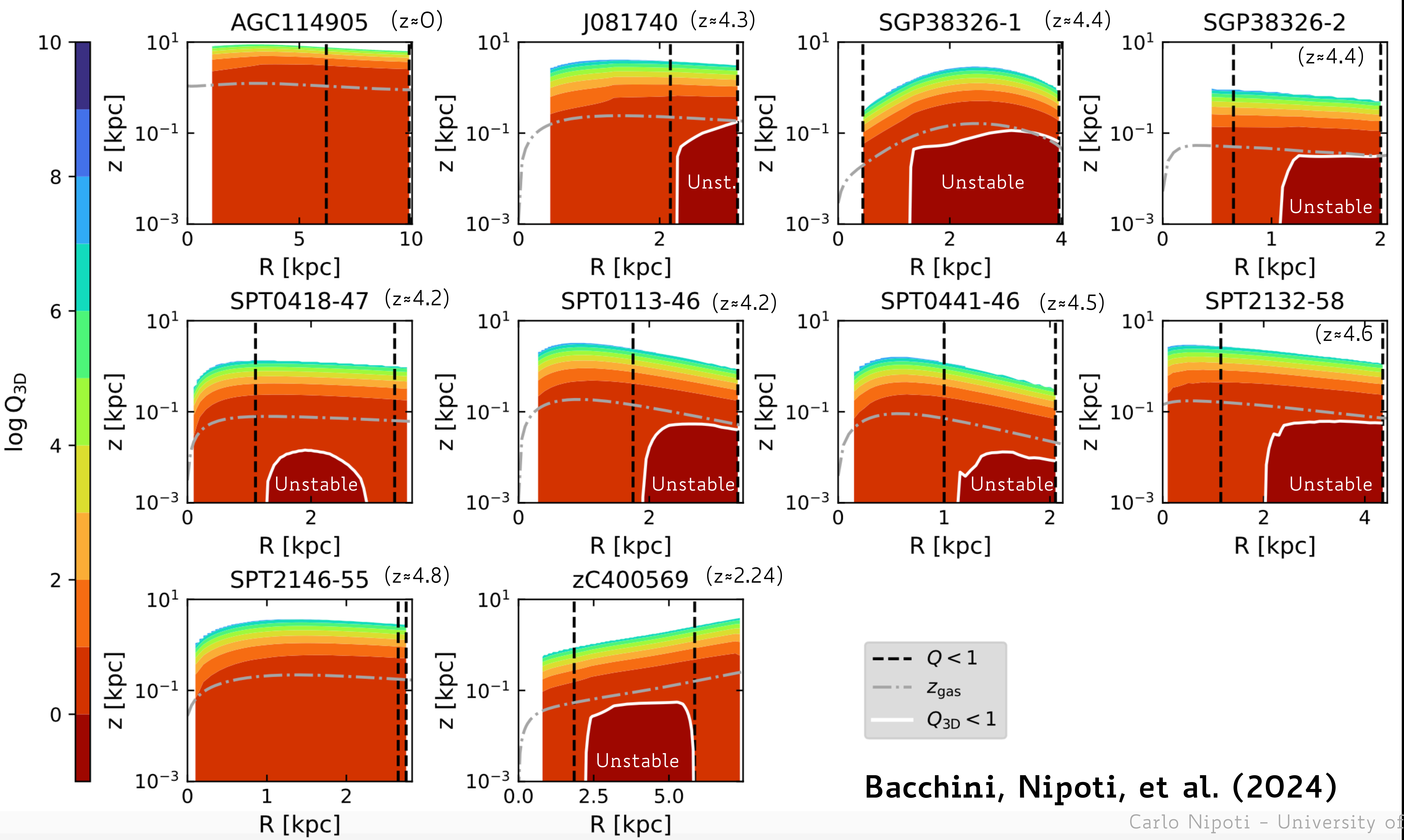


at given R

- $F_{\rm gas+\star}(k_R) > 1$ for instability
- $F_{\rm gas+\star} = F_{\rm gas} + F_\star$

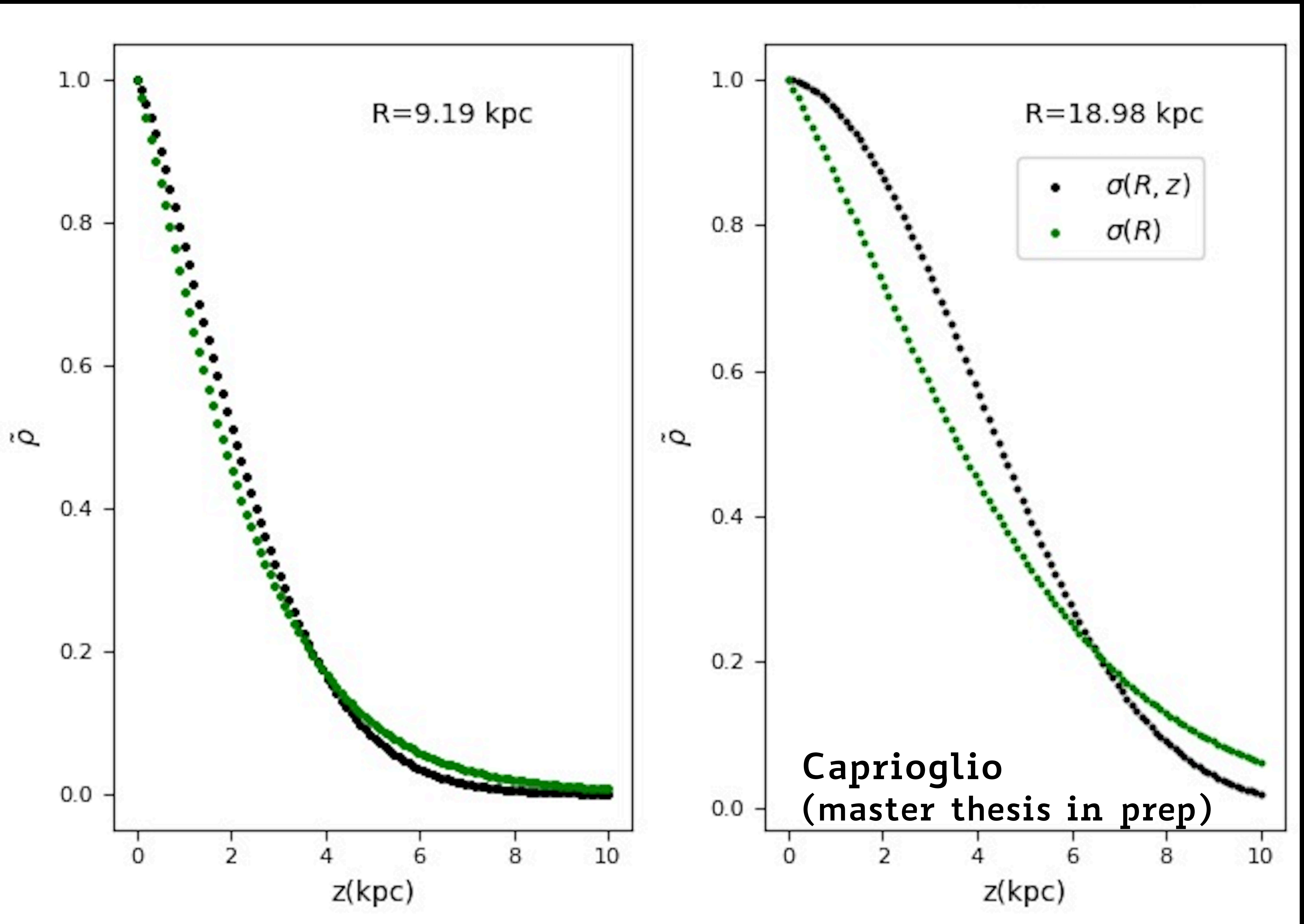


$Q_{3D}(R, z)$ maps for galaxies with $Q < 1$ (see Cecilia's talk)



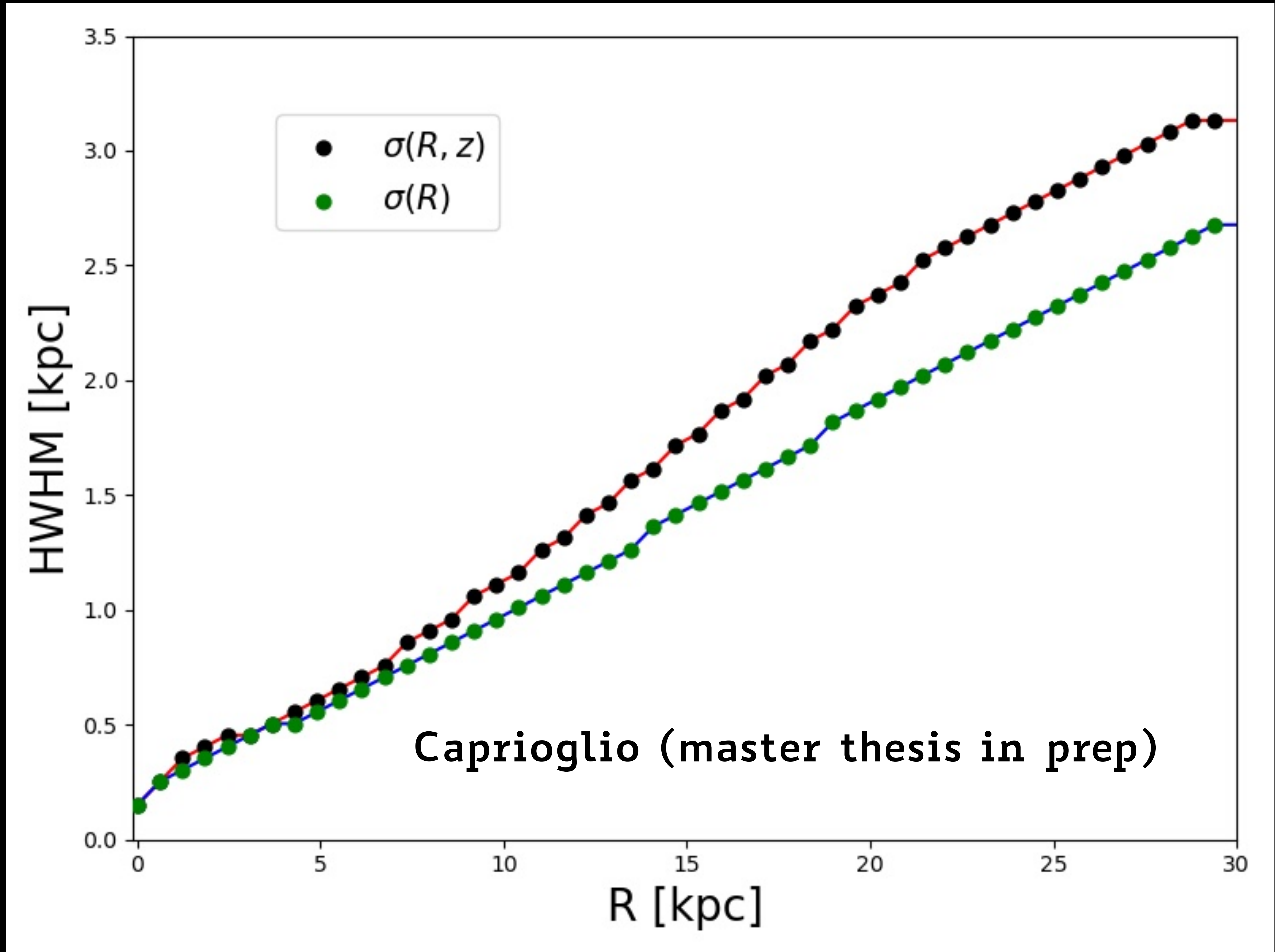
Bacchini, Nipoti, et al. (2024)

Full disc models with non-isothermal vertical profiles



(Caprioglio, Iorio & Nipoti in prep)

$$\sigma(R, z) \propto e^{-R/a} e^{-z/b}$$

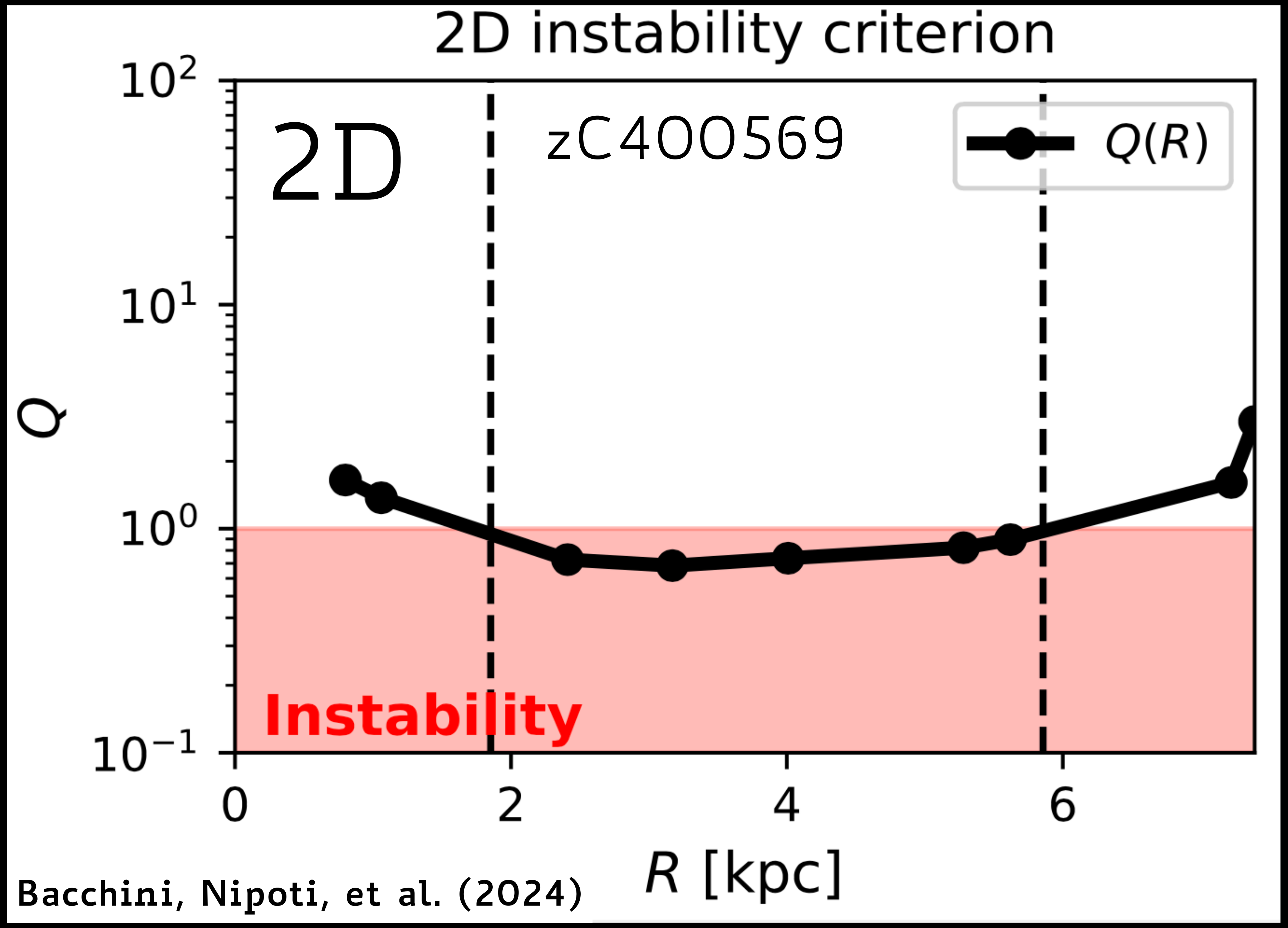


Modified version of the code GALPYNAMICS
(Iorio 2018)

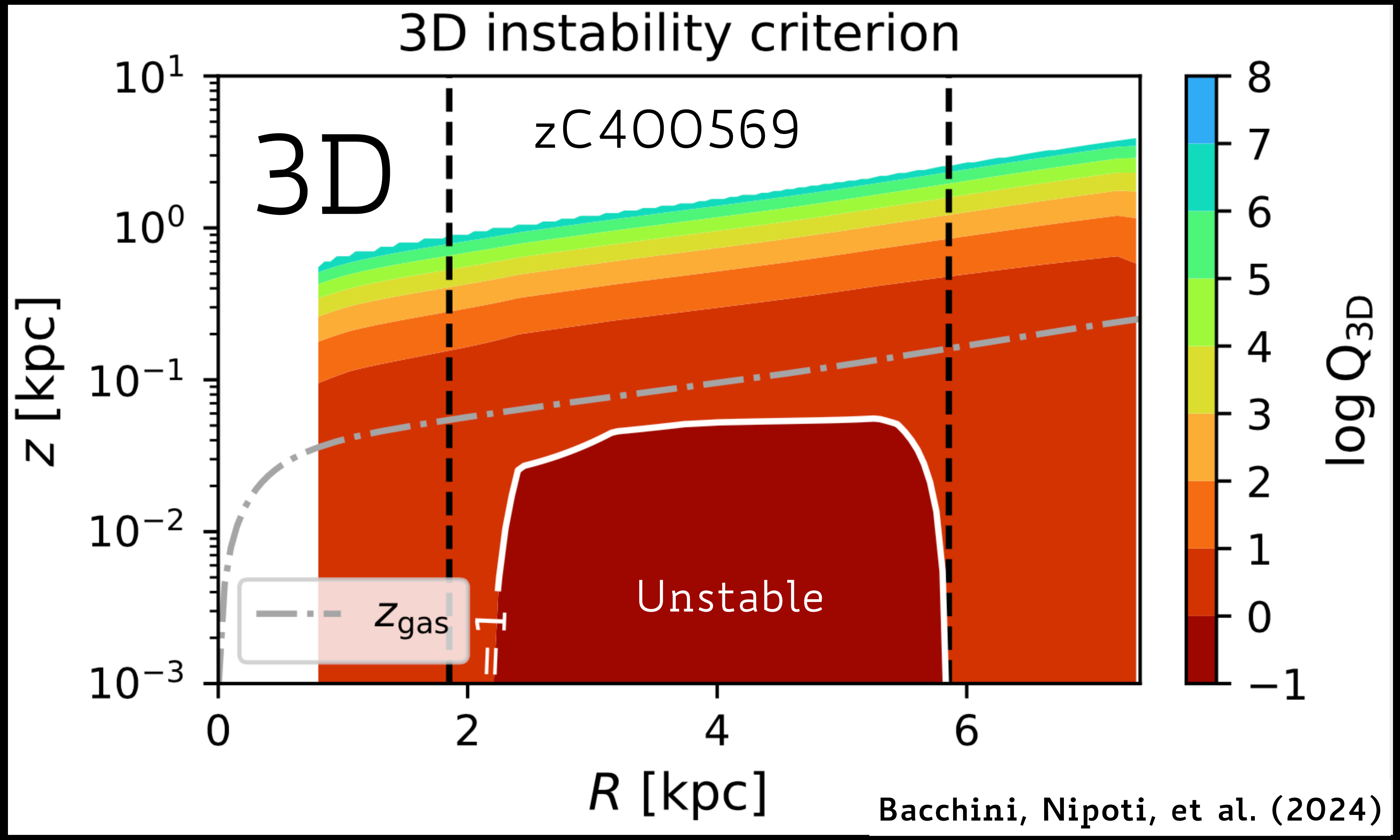
Conclusions

- 3D local gravitational instability analysis of thick discs is feasible
- $Q_{3D} \equiv \frac{\sqrt{\kappa^2+\nu^2}+\sigma h_z^{-1}}{\sqrt{4\pi G\rho}} < 1$ sufficient for instability

$Q_{\rm gas} = Q_{\rm gas}(R),\ Q_\star = Q_\star(R)$



$Q_{\rm 3D,gas} = Q_{\rm 3D,gas}(R,z),\ Q_{\rm 3D,\star} = Q_{\rm 3D,\star}(R,z)$



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Thanks!